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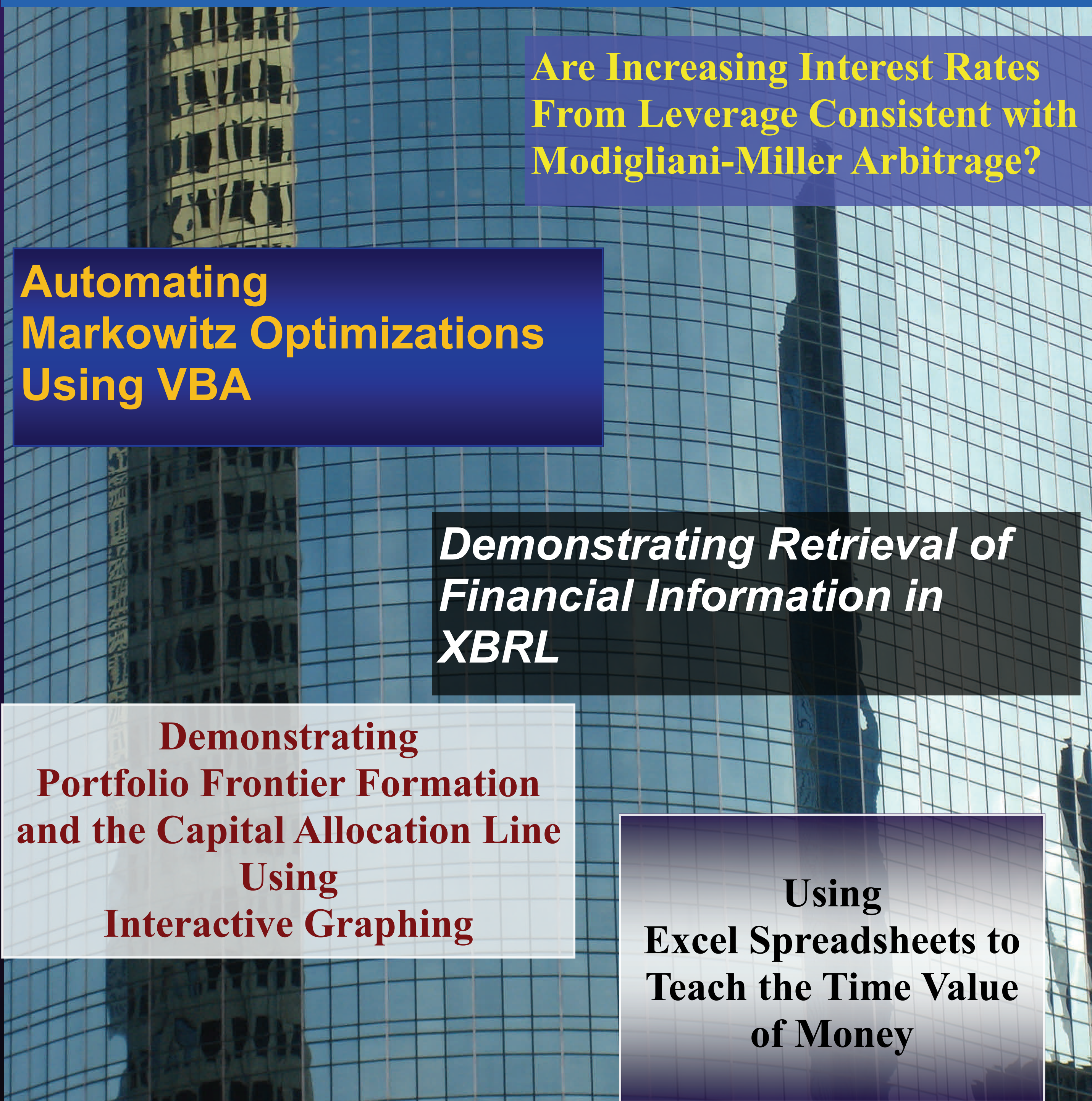

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COVER PHOTO: Wells Fargo Plaza, Houston, Texas

Are Increasing Interest Rates From Leverage Consistent with Modigliani-Miller Arbitrage?

Diane M. Lander and Glenn N. Pettengill

Among the most important contributions to the finance literature is Modigliani and Miller's seminal work on capital structure. As still taught in virtually all corporate finance textbooks, their famous Propositions I and II provide the foundation for the analysis of capital structure. Proposition I argues that under certain assumptions capital structure is irrelevant to the value of the firm. Proposition II describes the change in the required return to equity in response to a change in financial risk due to a change in leverage. In establishing Proposition II, Modigliani and Miller rely critically on an arbitrage process that forces the cost of equity to assume a value consistent with Proposition I. Although most of their analysis assumes constant debt rates, Modigliani and Miller allow that interest rates do vary with the level of leverage but that this variation does not affect the arbitrage process. In this paper, we argue that the arbitrage process is no longer dependable when debt rates change due to variation in leverage and discuss implications for teaching capital structure.

INTRODUCTION

Modigliani and Miller (1958[1]) establish two propositions concerning the capital structure decision, both of which are widely presented in textbooks as the foundation from which more advanced topics are examined. Proposition I argues that, under conditions of a perfect capital market (Table 1), capital structure has no impact on the value of the firm (p. 268[2]), implying that financial managers may rely on any mixture of debt and equity to finance the firm. Stated alternatively, the use of leverage has no effect on a firm's cost of capital (Figure 1). This relationship will be enforced by arbitrage opportunities between firms having the same level of business risk.

Leverage does, however, affect the firm's cost of equity (p. 268). Taking Proposition I as true, Proposition II states that the cost of equity linearly increases as leverage increases, reflecting the additional financial risk to the shareholder. Further, Proposition II states that the increase in required return for the equity stakeholder must be in direct relationship to the debt to equity ratio (p. 271). These relationships are summarized by Figure 1 and in equations (1) and (2).

$$\text{WACC} = kd * (D/V) + keL * (E/V) \qquad (1)$$

$$\text{Where} \quad keL = keU + (keU - kd)*D/E, \qquad (2)$$

The variables used in the equations (1) and (2) include the following:

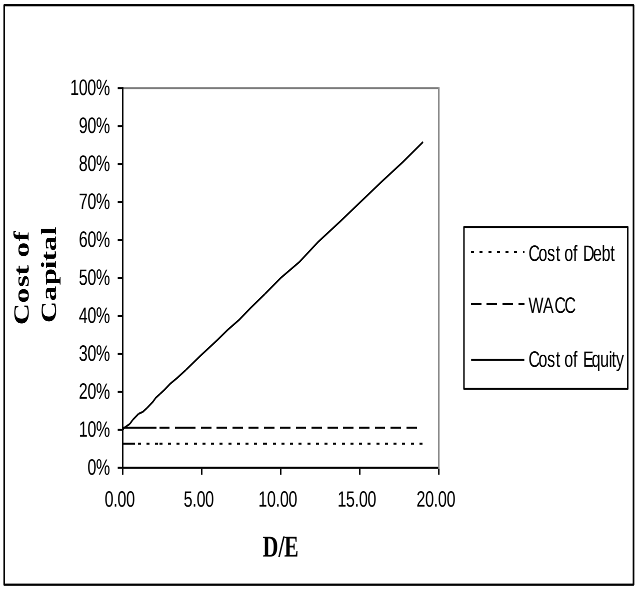
keL is the company's levered cost of equity.
keU is the company's unlevered cost of equity.
kd is the constant cost of debt.
D is the market value of the company's debt.
E is the market value of the company's equity such that D+E is the market value of the company.
WACC is the company's average cost of capital.

Modigliani and Miller argue that a number of scholars had developed arguments similar to Proposition I but that their development of Proposition II represents a departure from previous works (p. 271). Previous economic theory, they argue, developed the cost of capital with a "cavalier" treatment of uncertainty (p. 263). Paradoxically, although they investigate the impact of risk from the viewpoint of the equity investor, they appear to be themselves a bit "cavalier" with regard to the impact of risk on the debt

Table 1: Perfect Capital Market Assumptions

One possible set of assumptions related to Modigliani and Miller's paper is:
● Asset markets are frictionless (no transaction costs and information is costless and simultaneously available to all investors (i.e., no asymmetric information, no agency costs)).
● There are no market imperfections such as taxes (personal, corporate), regulations, and bankruptcy costs.
● The law of one price holds.
● Firms can be divided into risk classes based on business risk (volatility of operating income).
● There is one debt interest rate, investors and firms can borrow and lend at this rate, and all bonds are perpetuities.
● Operating earnings are not affected by leverage and are modeled as an expected no growth perpetuity (all profits are paid to the shareholders).

Figure 1 (Source: Lander and Pettengill (2010, p. 38)[3])



investor. With regard to equity holders, Modigliani and Miller argue that the unlevered required return will vary with the risk level of the operating income (p. 267)[4,5] and that leverage will additionally affect the variability of the equity returns within classes of risk given operating income (p. 268). The exact response to an increase in risk from leverage on the required return for the shareholder is determined by Proposition II. Modigliani and Miller introduce an arbitrage process that is critical in supporting Proposition II and is described in more advanced corporate finance texts. This arbitrage process is the focus of our paper and is reviewed and examined below. We find that the process is not dependable when interest rates increase due to increased leverage.

Unlike their treatment of risk for the equity holder, as Modigliani and Miller develop Propositions I and II, they assume that all debt issues provide constant and certain income streams, as well as yield the same rate of return[6], and the certainty of the debt income stream is not affected by the issuer (e.g., whether debt issued by a firm to purchase physical assets or debt issued by a household to buy shares) (p. 268). By implication, they assume the required return of the debt holder does not vary with the creditworthiness or the amount of leverage of the debt issuer.

After establishing Propositions I and II, Modigliani and Miller consider the impact of three qualifications. First, the impact of corporate taxes when interest payments are tax deductible. Second, the impact of the possibility that bond interest rates change with the issuer's credit worthiness and degree of leverage. Third, the impact of market imperfections that might interfere with the arbitrage process. Modigliani and Miller argue that, given certain additional assumptions, these qualifications do not affect their conclusions (pp. 272-281). In their 1958 paper, they conclude that taxes have no effect on Proposition I. They correct this position in their famous 1963 paper. As for the impact of multiple bond rates, they argue that, as long as the interest rate function is the same for all borrowers, which they suggest is a reasonable assumption, Proposition I continues to hold as is and Proposition II changes only in that the relationship between the cost of equity and leverage is no longer strictly linear (p. 273). The impact of market imperfections is dismissed with the argument that these imperfections will be neither large nor systematic (p. 281).[7]

Our focus in this paper is on the second qualification, the impact of the possibility that bond interest rates change with the issuer's credit worthiness[8] and degree of leverage. We examine the impact of relaxing this assumption on the arbitrage process supporting the Modigliani-Miller propositions and discuss teaching implications of the result. The rest of the paper is organized as follows. The next section deals with the impact of relaxing the assumptions made about the impact of the cost of debt on Propositions I and II. We provide a numerical example that illustrates the results and may be suitable for classroom use. The subsequent section presents a way to teach capital structure that allows for a smoother transition from the more theoretical Modigliani-Miller propositions to the more "real world" tradeoffs. The last section offers a brief conclusion.

RISK, LENDERS' REQUIRED RETURNS, AND THE MM PROPOSITIONS

The Modigliani-Miller (MM) propositions are developed with the presumption that the required return for equity investors varies with regard to both business risk and financial risk, but that the required return for lenders is not. Ignoring micro-structure effects of debt, three key issues challenge this assumption. First, just as equity investors will demand differing levels of return as business risk varies so should bond investors. Second, similarly, lenders should demand different levels of return depending on the credit standing of borrowers. This assertion directly challenges the assumption made in the Modigliani and Miller arbitrage argument that individual investors may borrow at the same rate as corporations. Third, a lender's required return will surely vary with the borrower's level of leverage. We investigate these in the order presented above and analyze each with respect to the assumption that the same debt interest rate applies across firms and individuals. When relaxing an assumption provides a critical challenge to the MM propositions, we developed arguments as to why and explore the teaching implications of our arguments.

The first challenge to a constant debt interest rate is that the required return for bond holders will certainly differ across firms with differing business risk. Corporations whose bond issues are provided with an AA rating borrow at a different rate than do corporate issuers whose bonds are given a BB rating. Recognition of this difference in lending rates does not, however, affect the development of the MM propositions. Modigliani and Miller recognize this difference with regard to equity investors and develop their arbitrage proof with the caveat that the arbitrage occurs within risk classes. Thus, relaxing this assumption only impacts their proof in terms of scale.

The other two challenges to the assumption of a constant debt interest rate provide more critical challenges to the arbitrage process that establishes the two MM propositions. If individual equity investors cannot borrow at the same rate as the corporation because lenders view them as inherently more risky or if different degrees of leverage create different required returns to debt, arbitrage is limited. As to the latter, Modigliani and Miller state that both theory and empirical observations indicate that borrowing rates tend to increase with the level of leverage (p. 273). They argue, however, given all borrowers face a common yield curve, Proposition II is impacted by this reality but Proposition I is not. In the remainder of this section, we first examine the validity of the presumption of equal borrowing rates and then examine the validity of the argument that varying borrowing rates due to leverage do not affect the validity of Proposition I.

The first question is: Can individual equity investors borrow at the same rate as can the corporation, as assumed when Modigliani and Miller develop their arbitrage argument? Modigliani and Miller defend this assumption on the basis that the rate on brokers loans, which is the rate at which arbitragers would borrow, had been similar to the rate on corporate loans (p. 274). Brealey, Myers and Allen (2008, p. 486) also defend this assumption, arguing that both home mortgage rates offered to individual home buyers and call rates on margin buying offered to individual investors compare favorably with rates paid by corporate bond issuers. Home mortgage rates, however, involve substantial collateral obligations on the part of the borrower. A more relevant question is whether the call rate at which equity investors would indeed borrow for purposes of equity investing is commensurate with the corporate lending rate. This comparison is also hampered with micro-market effects. The call loan requires margin. And the call loan is, well, callable. Thus, these rate comparisons are not sufficient to establish equality in borrowing rates. Individual equity investors cannot presume to borrow at the same rate as the corporation and, thus, the arbitrage process is not guaranteed.

The question then becomes: Are rates sufficiently different to make arbitrage implausible? When students are asked if they can borrow at the same rate as, say, IBM, they inevitably answer no, but this response misses the point. Some large equity investors, as Modigliani and Miller argue (p. 274), unlike most corporate finance students, may borrow at rates not substantially different than corporate rates. Indeed, some large equity investors may have higher credit standing than the corporations in which they are investing. One may argue, however, that equity investors should not be able to borrow at a lower rate to invest in the assets of a corporation than can the corporation itself. The argument that an equity investor can borrow at a rate sufficiently low as to provide arbitrage may not be certainly true, but it does not seem on the face to be a false assertion that the marginal equity investor can borrow at a sufficiently low rate to undertake the arbitrage activity necessary to validate MM I. Thus, we now turn to the final challenge to the arbitrage proof and the central issue of our paper, the impact of the amount of leverage.

In the remainder of this section, we develop the argument that the impact of increased borrowing rates due to leverage could prevent the arbitrage process developed by Modigliani and Miller and illustrate our argument with a numerical example. We assume throughout that individual investors can indeed borrow at the same rate as the corporation and all profits are paid to the shareholders.

We believe this numerical example is appropriate for classroom presentation and present the example in three stages. In the first stage, we show returns to equity holders in a levered and unlevered firm where results are consistent with Propositions I and II. In the second stage, we disturb the equilibrium posited by Propositions I and II and show how the arbitrage process developed by Modigliani and Miler corrects the market mispricing and restores equilibrium. In the third stage, we again disturb the Proposition I and Proposition II equilibrium but, in this stage, we assume that the leverage of the levered firm is sufficiently high such that the cost of additional debt increases for the levered firm.[9] We show how the arbitrage process, in this situation, then cannot re-establish equilibrium.

In all three stages of the numerical example, an instructor could postulate two firms, Firm A and Firm B, both of which fall into the same class of business risk and so have identical expected return distributions. An additional assumption that could be used in all three stages is that the expected annual return for both of these firms, as held by all investors, is 8% on assets of $1,000,000. The firms differ, however, in terms of their financing. Firm A is all equity financed. Firm B is financed only partially with equity. According to MM I, the firms must be priced to have equal value, and, if not, arbitrage activity would equate the value of the two firms. ROE, however, is different for the two firms, in agreement with MM II.

In stage 1 of the example, the instructor could illustrate the standard Modigliani-Miller equilibrium with the firms priced to have equal value. The instructor could postulate two consecutive years in which the firms get the same returns, as they should on average. Further, the example could assume that for both firms ROA is 10% ($100,000 profit) in the first year and 6% ($60,000 profit) in the second year, consistent with an expected return of 8%. The stage 1 example assumes market participants get it right: Firm A's equity is valued at $1,000,000; Firm B's equity is valued at $500,000. The total value of Firm B, including the $500,000 debt, is $1,000,000. Now the students could participate in calculating ROE.

For Firm A, ROE is easy. Given no interest payments, ROE is 10% in year 1 (100,000/1,000,000) and 6% in year 2 (60,000/1,000,000). For Firm B, additional work is required because of the leverage. And it may be well for the instructor to remind students by way of query that ROE is a percent determined by dividing the dollar returns available to the equity holders by total equity. For year 1, Firm B's ROE 13% ((100,000–0.07*500,000)/500,000), which is greater than Firm A's. The benefit of leverage! For year 2, however, Firm B's ROE is lower than Firm A's. Firm B's ROE is 5% ((60,000–0.07*500,000)/500,000). The two-edged sword of leverage! The average return to equity for Firm A is 8% and for Firm B is 9%. Firm B's equity investors get a higher return, on average, to compensate for the greater risk inherent with the leverage. The instructor could complete stage 1 by showing that the results obtained are consistent with Proposition II.

Table 2: Numerical Example Stage 1 Key Assumptions

- **All equity investors rationally price financial risk.**
- **Corporations borrow at a constant rate regardless of leverage.**
- **Firm A is all equity financed.**
- **Firm B is financed with 50% equity and 50% debt, and pays 7% for its debt.**
- **Individual investors can borrow at the same rate as the corporation.**
- **All profits are paid to the shareholders.**

$ROE_{Firm\ A}$ = keU + (keU – kd)*D/E
= 8% + 0% = 8% (3)

$ROE_{Firm\ B}$ = keU + (keU – kd)*D/E
= 8% + (8% – 7%) * (0.50/0.50) = 9% (4)

Table 3: Numerical Example Stage 2 Key Assumptions

- **Some equity investors irrationally price financial risk, but other equity investors will benefit from this irrational pricing through arbitrage.**
- **Corporations borrow at a constant rate regardless of leverage.**
- **Firm A is all equity financed.**
- **Firm B is financed with 50% equity and 50% debt, and pays 7% for its debt.**
- **Individual investors can borrow at the same rate as the corporation.**
- **All profits are paid to the shareholders.**

In stage 2 of the example, the instructor could illustrate the Modigliani-Miller arbitrage process by assuming a disturbance to equilibrium. The instructor could assume that, at the end of year 2, Firm A is still valued at $1,000,000, commensurate with the expected return, but postulate that naïve equity investors behaved foolishly and overpriced the equity of Firm B because they observed the higher average return to equity and think there is a benefit to leverage. The example could assume that the value of Firm B's equity is irrationally priced at $600,000 while the value of its debt remains at $500,000.

Now the instructor could postulate an equity investor who owns 1% of Firm B and who has read and understands Modigliani and Miller. This investor knows that his stock is overpriced relative to Firm A, so he conducts an arbitrage. He sells his shares in Firm B and receives $6,000 from the sale (0.01*600,000), has a really great party with $1,000[10], and uses the remaining $5,000, along with $5,000 borrowed at 7%, to buy a 1% stake in Firm A. This action could be presumed to occur at the end of year 2, just before two more consecutive years of returns equal to those for year 1 and year 2. That is, both firms experience a 10% return on assets in year 3 and a 6% return on assets in year 4. With this 1% stake in Firm A, this astute arbitrageur will get $1,000 in year 3 and pocket $650 after paying off the hard-nosed lender who demands her $350. Likewise he will pocket $250 (600–350) in year 4. Now other equity investors in Firm B catch on, and also sell their shares and buy shares in Firm A until equilibrium is restored with Firm B's equity again priced at $500,000.

Next, ignoring the problem that the arbitrage process might overprice Firm A[11], the instructor could posit a second investor who is considering buying 1% of the equity of either Firm A or Firm B and then ask the students which investment would be preferred? The higher average return to equity of Firm B or the lower risk of Firm A? The example shows that, regardless of his preference, the investor is indifferent between the two. If he prefers the tradeoff in Firm B, he can borrow $5,000 at 7%, use $5,000 of his own money, invest the $10,000 in Firm A, and receive exactly the same payoff as if he invested $5,000 in Firm B. Likewise, if he prefers the safer return of Firm A, he can lend $5,000 at 7%, invest $5,000 of his own money in Firm B, and receive exactly the same payoff as if he had invested $10,000 in Firm A. By borrowing or lending different amounts, this investor can achieve an infinite number of risk return tradeoffs using either firm. The end result is that investors are indifferent between owning 1% of Firm A or 1% of Firm B and the total value of Firm A and Firm B must be the same.

Thus far the example has supported the validity of the MM propositions. But now the example should continue by postulating a different equilibrium disturbing event. In the later part of their paper, Modigliani and Miller postulate that interest rates likely increase as leverage increases, but they argue that there is no basic change to Proposition I as the arbitrage is still alive. Investors just have to face a common yield curve with the firm (p. 273). But there is the problem. The common yield curve assumes that both the investor and the firm are at the exact same place with respect to their D/E ratios. But, if the required return to debt changes with the D/E ratio, the arbitrage falls apart, and, as the example continues, the students will see why.

In stage 3 of the example, the instructor could postulate that Firm B extends leverage to a total debt of $600,000 (lowering total equity to $400,000) with the last $100,000 in debt issued at a higher rate of interest, say 7.5%. The instructor may want to point out that, although the firm's D/E has changed, the income stream from operations remains the same. Given this assumption, the students could again help calculate ROE for a 1% investment in the equity of Firm B given two more consecutive years with ROAs of

10% and 6%, respectfully. For the next year, Firm B's ROE is 14.375% = ((100,000–0.07*500,000–0.075*100,000)/ 400,000).

Table 4: Numerical Example Stage 3 Key Assumptions

- **Some equity investors irrationally price financial risk.**
- **Corporations with higher levels of leverage borrow at higher rates.**
- **Firm A is all equity financed.**
- **Firm B is financed with 40% equity and 60% debt, and pays 7% for the first $500,000 borrowed and 7.5% for the last $100,000 borrowed.**
- **Individual investors can borrow at the same rate as the corporation.**
- **All profits are paid to the shareholders.**

For the following year, Firm B's ROE is 4.375% ((60,000 – 0.07 * 500,000 – 0.075*100,000) /400,000). The average return for Firm B is now 9.375%. More leverage, higher overall average return. No surprise here.

Continuing on, the example should now assume a third investor, Risky Ruby, who likes what has happened at Firm B. She thinks the increase in return to this average of 9.375% more than compensates for the added financial risk. She does not realize that 9.375% just adequately compensates for the increased level of financial risk. The question now is: Can she get that same return by investing in Firm A using 60% borrowed funds? The previous investors did using 50% borrowed funds when Firm B was 50% debt financed, and Modigliani and Miller assume she can, but now we have to make some serious assumptions about where she is on her yield curve. The following calculations will show the students that investors are no longer indifferent between investing in Firm A and Firm B. Personal leverage no longer assures the same return from the two firms.

There is no reason to assume that the previous investors and Ruby differ in terms of where they are in their levels of personal leverage. But, as long as the required return to debt changes as the level of personal leverage changes, we cannot be certain about the rate at which this risk-loving investor can borrow. The example could first ask what Ruby would earn from investing in Firm A if she did so by borrowing 60% of her invested funds at 7%, a rate reflecting her personal leverage. That is, if she borrows $6,000 at 7% and also invests $4,000 provided by her Uncle Harry, what is her return on the gift money from Uncle Harry?

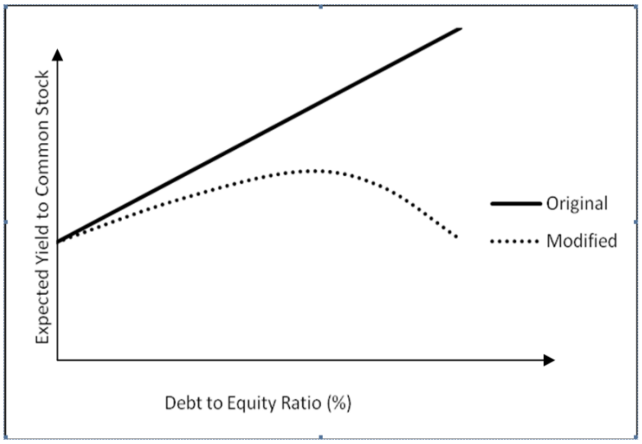
To show her return, the instructor could use two more years of a 10% ROA followed by a 6% ROA. ROE calculations show Ruby's return in Firm A to be 14.5% in the first year ((1,000–0.07*6,000)/4,000) and 4.5% in the second year ((600–0.07*6,000)/4,000). In both years, her return from investing in 1% of Firm A would be higher than the return from investing in 1% of Firm B. She is no longer indifferent between investing in Firm A and investing in Firm B. She may have the same leverage in her investment in Firm A as Firm B, but she can still borrow at 7% because her personal leverage is different from the leverage of Firm B. She prefers investing in Firm A, and, if she is a typical investor, as is implicitly assumed in the arbitrage example, values for Firm A and Firm B will not be equalized. The market is saying that Firm B is too levered for a 7% cost of debt.

So far, the example has shown that if Ruby could borrow at 7% because her personal leverage allowed her to borrow at that rate, a rate less than paid by Firm B, she is no longer indifferent between Firm A and Firm B. Further, if other investors can duplicate Ruby's investing, these two firms will not have the same value. Firm B suffers from too much leverage. The stage 3 example could finish by considering the possibility that Ruby's lender is concerned with the percentage of the investment funds that she borrows (60%) and charges her accordingly. If, at the margin, this risk requires a 7.5% rate, as for Firm B, that is the rate she gets. Now what would Ruby's return look like if she was to borrow $6,000 at 7.5% and also use the $4,000 gift money to invest in Firm A? The instructor could walk the students through the calculations one last time. Her return would be 13.75% in the first year ((1,000–0.075*6,000)/4,000) and 3.75% in the second year ((600–0.075*6,000)/4,000). This is definitely not a good plan. Ruby would earn a lower average return but with the same level of financial risk. If she wants more leverage than her investment in Firm A would provide, she best directly invest in Firm B. Again, the values of the firms are affected by the amount of leverage used because market borrowing rates change with the amount of leverage.

The only way Ruby is indifferent between investing in Firm A and investing in Firm B is if the lender charges her 7.083%, the weighted average of the rates for Firm B's two debt issues. But will the market lend to Ruby at 7.083%? The lender will charge Ruby either 7% based on her personal leverage or 7.5% based on the percentage of the investment funds that she borrows (60%) to invest in Firm A, which itself is 0% debt financed. If the market will not lend to her at 7.083%, and there seems to be no rational reason for it to do so, arbitrage may not re-establish equilibrium, MM I may no longer hold, and, thus, MM II may not adjust as Modigliani and Miller suggest.

We can look at the impact of higher borrowing rates from higher leverage by examining their impact on the cost of equity. Modigliani and Miller develop their propositions assuming that all debt issues "must yield the same rate of return" (p. 268). As noted early on, after developing their propositions, they consider three qualifications, one being that there are multiple debt interest rates. They state that both theory and empirical observations indicate that borrowing rates tend to increase with the level of leverage (p. 273). But then they argue that, as long as all borrowers face a common yield curve, Proposition I maintains and Proposition II changes in that the required return to equity for the levering firm that suffers an increase in the cost of debt tends to increase at a slower rate or even fall (p. 275) (Figure 2)12. Proposition II must change in this way because of Modigliani and Miller's assertion that arbitrage can still work and, as a result, the overall cost of capital will be constant.

Figure 2: MM Proposition II in D/E Space with Multiple Debt Interest Rates



Source: Figure 2 above approximates Modigliani and Miller's Figure 2 (1958, p. 275).

According to Modigliani and Miller, the relationship between the Expected Yield to Equity and the Debt to Equity Ratio (%) could be represented by the modified curve, "although in practice the curvature would be much less pronounced" (p. 275).

But how can it be that the required return to equity falls with an increase in leverage? Returns to equity decreasing is even somewhat inconsistent with Modigliani and Miller's own statement that the required return to equity tends to increases with an increase in the use of leverage. Modigliani and Miller do provide an explanation, but it is contingent on arbitrage holding. They say, "Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve MD, this demand would be reinforced by the action of arbitrage operators." (p. 276)

Yet the numerical example presented above demonstrates that arbitrage will not always be able to restore equilibrium. Moreover, the required return to equity cannot decrease with increased leverage because financial risk increases with greater leverage. And, if financial risk increases with greater leverage, then, in response to an increased D/E, the required return to equity could decrease only if that higher D/E suddenly had less financial risk. Logically, just the opposite will be true. In the case of increasing required returns to debt at high levels of D/E, the risk of bankruptcy will adversely affect equity holders as well. Witness the recent extreme fall in the price of stocks, reflecting higher required returns, as firms in the automobile industry approached bankruptcy.

Our last example clearly shows that leverage matters if interest rates change with the D/E ratio, that Proposition I, as well as Proposition II, may be impacted by this reality, and, if so, required returns to equity likely will not fall but increase, maybe even at an increasing rate. Now the question is how to relate this result back to Modigliani and Miller's original work? How would they have represented our last example? A good guess is that they would have said that Firm B has $6,000 of debt at 7.083%. What is different in our example is that there are two issues of debt, each at a different rate, and so two perfect market assumptions are relaxed. First is the assumption of one constant and certain rate to debt. But second is that firms have only one issue of debt. This combination of multiple issues each at a different rate, which certainly is "real world," leads to the results above.

TRANSITIONING FROM MM TO MORE "REAL WORLD" CONSIDERATIONS

Modigliani and Miller argue that increases in the required return to bond holders affects Proposition II but not Proposition I. We argue above that both propositions can be impacted when the cost of debt is allowed to vary. If one accepts our argument, then obviously there exist implications for teaching capital structure theory and practice. We suggest that recognizing the relationship allows for a smoother transition from the more theoretical MM propositions to the more "real world" tradeoffs.

In our experience, traditional finance textbooks that present the MM propositions typically transition to "real world" applications with somewhat of a disconnect. One example is Brealey, Myers and Allen (2008). Chapter 18, which asks the question "Does Debt Policy Matter,?" presents the Modigliani and Miller concepts, including showing a non-linear increase in the required return to equity as leverage becomes high, and concludes that "debt policy rarely matters in well-functioning capital markets with no frictions" (p. 496). It seems a bit incongruous, therefore, that the next chapter, Chapter 19, is titled: "How Much Should a Firm Borrow?" It is not surprising that Chapter 19 does not include any of the cost of capital graphs used in Chapter 18 to present the Modigliani-Miller foundation, but students may wonder why they have studied Modigliani and Miller in Chapter 18 if there is no application when they study the question "How Much Should a Firm Borrow?" in Chapter 19.

In an introductory corporate finance course, depending on the enrollment, the instructor may not want to introduce the Modigliani-Miller concepts when discussing capital structure. We suggest, however, that presentations of capital structure and cost of capital in upper division and graduate courses should indeed begin with MM I and MM II and that advanced students be provided an exercise, such as the numerical example above, showing the standard arbitrage opportunities when levered and unlevered firms are not valued equally, thus allowing the instructor the option to emphasize the concepts of financial risk for the equity investor and increased variability in returns to the equity investor as leverage increases.

After the arbitrage and financial risk concepts are discussed, the instructor may consider relaxing the key assumptions regarding debt and taxes. An exercise, such as the one in this paper, could be used to illustrate that arbitrage can fail when borrowing rates change with the degree of leverage. This may then be followed with an exercise to show the impact of taxes. Combining these two influences allows for the presentation of the tradeoff theory with graphical presentations showing the influence of both taxes and increasing interest rates as the required return to bond holders increases with increasing leverage and the chance for default.

The instructor may wish to take the opportunity to contrast the changes in the required return to equity and required return to bond holders as leverage increases. Does the required return to bond holders stay virtually the same over some range as leverage increases when the probability of default is quite low? If this is the case, can the same principle apply to equity holders? In the absence of arbitrage enforcing a particular relationship between the required return to equity and the debt to equity ratio, might not initial increases in the required return to equity holders be more moderate than under MM II because the investors do not notice the change in financial risk?

Such discussions could also facilitate micro-market examinations. To what extent can an equity investor borrow at the same rate as the corporation? Would arbitrage ever take place between levered and unlevered firms with similar levels of business risk? Or, are such differences insufficiently transparent for the operation of arbitrage in the "real world?" All of these discussions easily follow from a presentation of the MM propositions and allow for a guided transition to decisions that the students, when financial managers, will indeed make.

CONCLUSION

For over fifty years, Modigliani and Miller's seminal work has stood as the starting point for the analysis of capital structure. Their famous Proposition I argues that, under certain assumptions, capital structure is irrelevant to the value of the firm and the use of leverage has no effect on a firm's cost of capital. Modigliani and Miller argue that this proposition stands even given the favored treatment of debt under the tax laws and increasing borrowing rates due to the possibility of financial distress as leverage increases. In their 1963 paper, they acknowledge that corporate tax laws do invalidate Proposition I. In this paper, we examined the impact of increasing borrowing rates due to leverage on the validity of Proposition I and argue that increases in leverage and the accompanying changes in the required returns to bond holders impact both Proposition I and Proposition II and influence the optimum capital structure. We suggest that recognition of this relationship should influence the teaching of capital structure because the presentation of capital structure theory in standard finance textbooks may inadequately bridge the Modigliani-Miller foundation and more "real world" considerations.

The authors thank participants of the 2010 Financial Education Association Conference and the 2012 Academy of Economics and Finance Conference for their comments. Lander acknowledges financial support from Saint Michael's College Faculty Development funding and the Office of the VPAA.

END NOTES

1. Unless otherwise noted, references to Modigliani and Miller's paper are to their 1958 paper.
2. Page references to selected statements are given to assist the reader in finding the original statements in the 1958 Modigliani and Miller paper.
3. Lander and Pettengill (2010) provide a review and numerical examples and show that, although the cost of equity is linear in D/E space, it is not linear in Debt Ratio space and that, in Debt Ratio space, the cost of equity increases rapidly at high levels of debt in accordance with MM I and MM II. They argue that most students and even faculty and practitioners think in terms of Debt Ratio space and may not notice that presentations such as shown in Figure 1 are in D/E space. This may allow the mistaken assumption that the cost of equity grows linearly with increasing debt ratios.
4. Modigliani and Miller (1958) define risk classes as being similar to industries. They say:
We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated wnth) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the ratio of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. (p. 266)
5. Recently Ghosh and Ghosh (2010) explore the impact of relaxing the assumption of constant operating income on Proposition I and II. We explore the Modigliani-Miller propositions leaving this assumption intact.
6. Modigliani and Miller's assumption about this debt interest rate is not clear. In some places, they assume this is the risk-free rate and, in others, they assume this is a rate commensurate with the business risk of the firm.
7. This argument is more extensively developed in Modigliani and Miller's 1959 reply to Durand (1959). Much of the discussion centers on margin and short-selling restrictions.
8. We do not purport to be the first to raise the issue concerning the equality of borrowing rates. Rose (1959) raises the issue but concentrates on the equality of share ownership in levered and unlevered firms, largely missing the argument developed as Proposition II.
9. In a classic article, Baxter (1967) explores the role of increased risk with leverage, but not from the viewpoint of conducting arbitrage. Rather, Baxter (1967) argues that the risk of ruin changes the assumption that the levered and

unlevered firms enjoy the same stream of operating income. The increased chance of bankruptcy decreases the expected cash flow to the levered firm due to expected bankruptcy costs. Thus, Baxter (1967) develops the tradeoff between the tax advantage and bankruptcy costs in determining the optimum level of leverage.

10. In their arbitrage example, Modigliani and Miller do not allow the investor in the overvalued firm to have a party. Instead the investor dutifully invests all of his proceeds from the shares of the levered firm into the purchase of the unlevered firm, borrowing enough to own a greater percentage stake in the unlevered firm. Durand (1959) in his examination of the arbitrage process allows the investor to keep excess funds. He simply does not specify the use of these funds. We agree with Durand that the investor in the levered firm disgorges his excess cash for two reasons. First, students rather appreciate the ability to have a really great party. Second, by investing only part of the proceeds, the investor from the unlevered firm has a debt to equity ratio equal to that which was originally established by the levered firm and which the levered firm maintains in equilibrium. The latter rationale is important to Durand's example.

11. In their example, Modigliani and Miller show equilibrium to be reestablished by selling pressure lowering the price of the overvalued levered firm. Elsewhere, they identify price changes in both stocks. In the former case, the question exists as to why buying pressure affects only one firm. In the other case, the question becomes, if the unlevered firm was originally priced to be in equilibrium, how would a new higher price also be in equilibrium? And what price shift would occur for all other firms of similar risk levels?

12. The authors wish to note that Figure 2 appears very similar to graphs depicting the tradeoff theory when corporate taxes and bankruptcy costs are jointly considered. Despite the similarity between the two graphs Figure 2 depicts a different concept altogether.

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Automating Markowitz Optimizations Using VBA

David Porter and Robert Stretcher

In the interest of reducing grading workload and managing cross-student sharing (cheating), we provide an example of automating the construction of Markowitz (1959) efficient frontiers. The program automates the entire procedure from downloading data for a specified number of securities (using either randomly generated securities from a provided list or specified tickers) to generating graphs of the efficient frontier. Many commercially available optimization programs are available, but this program is free to use and works within Microsoft Excel (2007-2010), the software package most professors use when creating optimization assignments. The program is written in Visual Basic for Applications (VBA) but does not require any knowledge of VBA to use. The program allows easy creation of random portfolios for student assignments so that cross-student sharing can be minimized without adding an additional grading burden on the professor or TA. The professor can also be assured that the stocks selected produce the desired type of optimization (actually work) before using each set of securities. Where students are allowed to select their own portfolios, the program allows professors to easily check computations and optimizations without having to manually optimize each portfolio. The program can also be used to demonstrate the similarity of efficient frontiers across different stocks, portfolio sizes, and dates.

INTRODUCTION

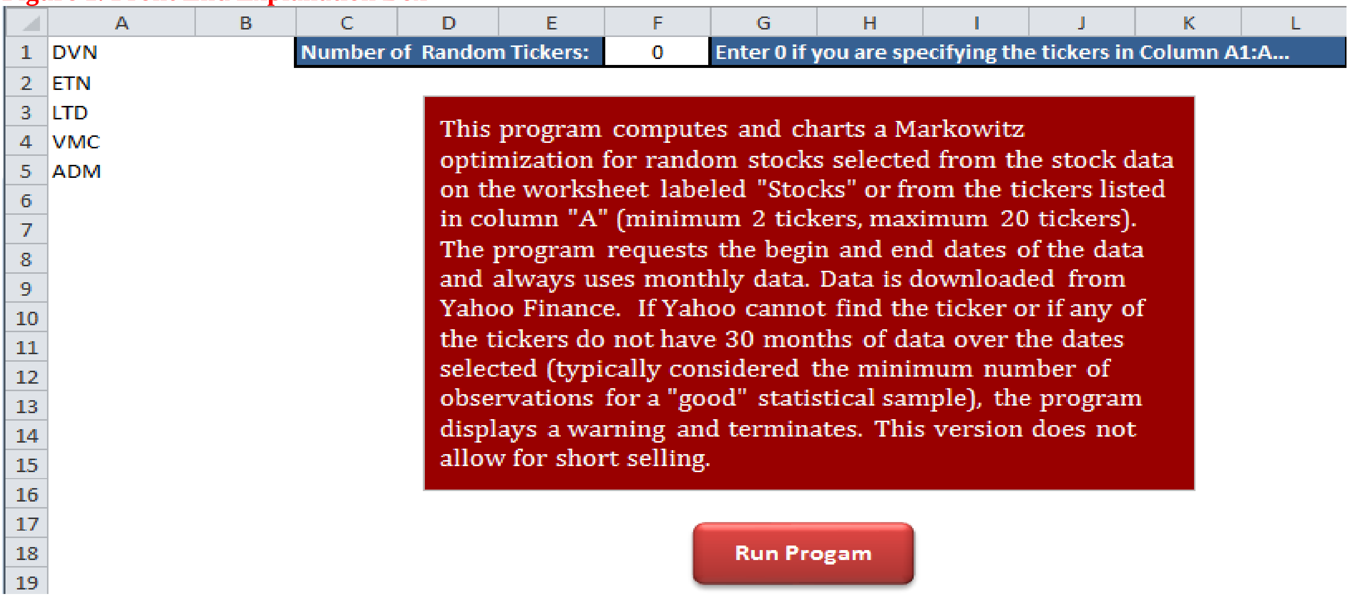
Cutbacks in government funding to public universities has resulted in reductions across many areas including but certainly not limited to the hiring of new faculty to keep class sizes small. As class sizes increase, grading of assignments becomes more problematic both in the amount of time required and in the possibility of cross-student sharing (also referred to as free-riding or cheating). In the case of investments courses, many professors require students to complete some type of optimization assignment. With class sizes exceeding 100 students, however, offering individual students the opportunity to select their own stocks makes the grading process difficult or prohibitive. As a result, students are commonly placed in groups, and entire classes may be assigned the same set of stocks to optimize. Although this technique significantly reduces the length of time needed to evaluate the assignments, it also increases the probability of cross-student or cross-group sharing making it difficult to determine which students actually understand the optimization process. One solution to these issues is to use an automated system designed to generate Markowitz (1959) optimizations so that every group or student could be either assigned a separate set of stocks to optimize, or be allowed to select their own securities. In this paper we describe a VBA program that can be used to automate the optimization of portfolios using Solver in Microsoft Excel. The object is not to teach VBA but to offer some typical code that can be used to expedite a professor's use of this type of automation.

DESCRIPTION OF THE "FRONT-END" OF THE PROGRAM

The program is designed for a personal computer (PC) running a recent version of Excel (2007 or 2010) with the Solver add-in installed and does not require any knowledge of VBA, unless the individual professor wishes to adjust the code for his/her own purposes. The program will not run on an Apple operating system but could work if the MAC was dual booted into Windows and was running the appropriate Excel version. Since the VBA code is essentially a large macro, when the file containing the program is opened, two warnings may occur: (1) This file originated from an Internet location and might be unsafe and, (2) Some active content has been disabled. Click for more details. In both cases, clicking "Enable Content" allows the VBA to run. The file will then default to the "Explanation" sheet that contains a brief explanation of the program.

As noted in the red box, the program downloads monthly data from Yahoo for either a randomly selected portfolio of securities or a specific set of ticker symbols. If the user wants random portfolios, the number of securities is entered in cell F1. If the user wants a specific set of securities, the tickers should be entered in column A, starting with cell A1, and cell F1 should be set to 0. Cell F1 is checked first, so if it contains a number between 2 and 20, a random portfolio will be generated even if there are specific tickers in column A. The restriction of 2 to 20 tickers is usually sufficient for most assignments but can easily be altered by changing the VBA code. The program will compute and graph the Markowitz optimization assuming no short selling. Stocks with less than 30 months of data are not used since the mean return is used as the expected return and 30 observations is typically assumed to be a "reasonable" statistical sample. The user can also change the minimum observation assumption by altering the VBA code.

Figure 1. Front End Explanation Box



Before clicking "Run Program", the user should note the status bar at the bottom of the spreadsheet:

Figure 2. Status Bar.

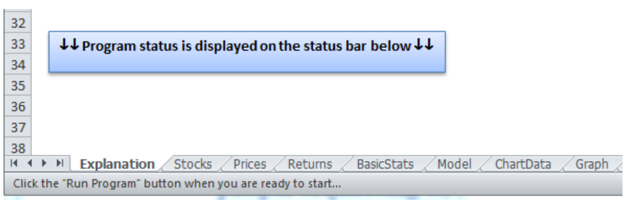


Figure 3. Prices Sheet.

	A	B	C	D	E	F
1		PCG	ETR	IFF	CTL	APC
2	Date	Adj Close	Adj Close	Adj Close	Adj Close	Adj Close
3	Aug-12	43.41	68.08	60.18	41.55	69.18
4	Jul-12	46.16	71.83	55.43	40.84	69.35
5	Jun-12	45.27	67.11	54.49	38.82	66.12
6	May-12	43.26	63.79	55.75	37.85	60.84
7	Apr-12	43.74	63.98	59.54	37.21	73.02
8	Mar-12	42.97	65.58	57.95	37.30	78.13
9	Feb-12	40.83	65.02	56.09	38.13	83.81
10	Jan-12	39.83	66.89	54.89	35.08	80.42
11	Dec-11	40.38	70.43	51.55	35.24	76.05
12	Nov-11	37.63	67.83	53.05	34.85	80.88
13	Oct-11	41.57	65.89	59.21	32.75	78.12
14	Sep-11	40.99	63.15	54.96	30.76	62.75
15	Aug-11	40.59	62.12	56.43	32.90	73.30
16	Jul-11	39.71	62.75	59.49	33.78	82.06
17	Jun-11	40.29	64.14	62.48	36.80	76.29

The status bar reports where the program is at any point in time. If it appears that nothing is happening, the user should check the status bar. Also note that the program generates six additional spreadsheets. Prices, Returns, BasicStats, Model, ChartData and Graph. The seventh additional sheet is the list of stocks used for randomly generating portfolios. This sheet can be populated by the user but the original file contains about 500 stocks as examples. The "Prices" sheet contains the adjusted closing prices downloaded from Yahoo. Since the program uses monthly prices, these prices are the last adjusted closing price of each month.

The "Returns" sheet contains the returns computations based on the downloaded prices. Where the number of prices differs across the securities, the returns are truncated to the shortest set of data. This is necessary for the computation of correlations and covariances. If the number of returns is less than 30 for any of the securities the sample is considered too small and the program will terminate.

The Basic Stats sheet contains summary measures for the stocks: means, standard deviations, minimum return, maximum return, and range of returns, as well as the correlation matrix.

Figure 4. Returns Sheet.

	A	B	C	D	E
1	PCG	ETR	IFF	CTL	APC
2	-5.9575	-5.2207	8.5694	1.7385	-0.2451
3	1.9660	7.0332	1.7251	5.2035	4.8851
4	4.6463	5.2046	-2.2601	2.5627	8.6785
5	-1.0974	-0.2970	-6.3655	1.7200	-16.6804
6	1.7919	-2.4398	2.7437	-0.2413	-6.5404
7	5.2412	0.8613	3.3161	-2.1768	-6.7772
8	2.5107	-2.7956	2.1862	8.6944	4.2154
9	-1.3621	-5.0263	6.4791	-0.4540	5.7462
10	7.3080	3.8331	-2.8275	1.1191	-5.9718
11	-9.4780	2.9443	-10.4036	6.4122	3.5330
12	1.4150	4.3389	7.7329	6.4694	24.4940

Figure 5. Basic Stats Sheet.

	A	B	C	D	E	F
1	Summary Measures for Stock Returns					
2						
3	Stocks	PCG	ETR	IFF	CTL	APC
4	Means	0.3894	-0.1912	0.7039	0.5886	1.4230
5	Std. Dev.	4.3698	5.8038	6.3021	6.8484	12.5459
6	Min	-9.4780	-13.9126	-19.2308	-31.4738	-30.8929
7	Max	8.6084	16.3804	16.7496	13.6035	36.2108
8	Range	18.0864	30.2930	35.9804	45.0773	67.1037
9						
10	Correlations	PCG	ETR	IFF	CTL	APC
11	PCG	1.0000	0.4871	0.3017	0.1946	0.2045
12	ETR	0.4871	1.0000	0.2949	0.4240	0.5072
13	IFF	0.3017	0.2949	1.0000	0.5141	0.4569
14	CTL	0.1946	0.4240	0.5141	1.0000	0.5042
15	APC	0.2045	0.5072	0.4569	0.5042	1.0000

The "Model" sheet contains the data needed for the optimization: the weights for each stock for the last optimization run, the expected returns (the means), the variance-covariance matrix, the expected return, variance and standard deviation for the optimized portfolio and the target cell. The target cell changes as the program generates the chart data from the minimum variance portfolio to the maximum return portfolio.

Figure 6. Model Sheet.

	A	B	C	D	E	F	G
1	Portfolio Selection Model						
2							
3	Stock	PCG	ETR	IFF	CTL	APC	Sum
4	Weights	0.0134	0.0000	0.4568	0.0000	0.5299	1.0000
5	Means	0.3894	-0.1912	0.7039	0.5886	1.4230	
6							
7	Covariances	PCG	ETR	IFF	CTL	APC	
8	PCG	18.7773	12.1475	8.1690	5.7267	11.0263	
9	ETR	12.1475	33.1226	10.6063	16.5712	36.3190	
10	IFF	8.1690	10.6063	39.0544	21.8195	35.5250	
11	CTL	5.7267	16.5712	21.8195	46.1188	42.5989	
12	APC	11.0263	36.3190	35.5250	42.5989	154.7774	
13							
14	E(Rp)	1.0807		Target	8.3102		
15							
16	Var(Rp)	69.0594		Sigma(Rp)	8.3102		

The "ChartData" sheet contains the Solver optimization output for each of the 16 runs from the minimum variance portfolio to the maximum return – the weights for each security for each run, as well as the portfolio expected return and standard deviation. The number of runs is arbitrary and 16 (15 Solver runs plus the maximum return portfolio) was chosen as a number that would produce a nice looking graph the majority of the time.

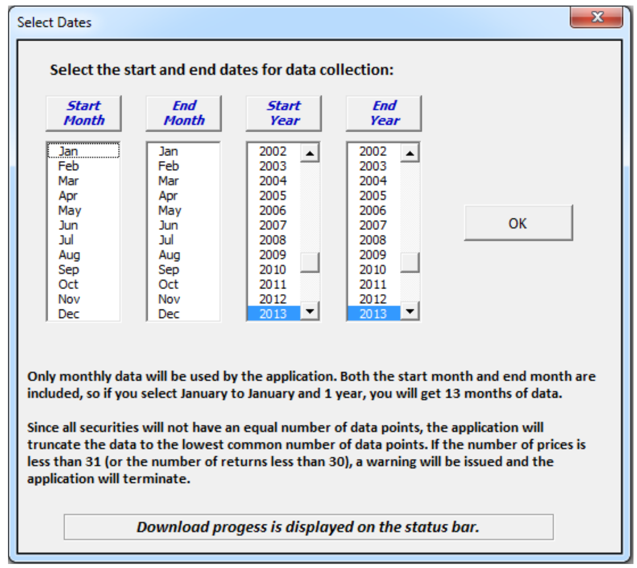
The "Graph" sheet contains a scatter plot of the chart data. Since most graphs of the efficient frontier look better with more points near the minimum variance point, the chart data is generated using that assumption. The graph will also auto-scale itself based on the data. An example of the graph output is in Figure 10.

Figure 7. ChartData Sheet.

	A	B	C	D	E	F	G	H	I
1	Efficient Frontier				Optimal Portfolio Weights				
2		Port. Std.Dev.	Port. Return		PCG	ETR	IFF	CTL	APC
3		12.5459	1.4230		0.0000	0.0000	0.0000	0.0000	1.0000
4		8.3102	1.0807		0.0134	0.0000	0.4568	0.0000	0.5299
5		6.1355	0.8569		0.2784	0.0000	0.3871	0.0000	0.3345
6		5.0190	0.7197		0.4213	0.0000	0.3268	0.0393	0.2125
7		4.4457	0.6318		0.5010	0.0000	0.2774	0.0887	0.1330
8		4.1514	0.5729		0.5542	0.0000	0.2443	0.1217	0.0797
9		4.0003	0.5323		0.5911	0.0000	0.2215	0.1445	0.0429
10		3.9227	0.5036		0.6169	0.0000	0.2055	0.1605	0.0170
11		3.8829	0.4831		0.6394	0.0000	0.1894	0.1712	0.0000
12		3.8624	0.4607		0.6465	0.0235	0.1661	0.1638	0.0000
13		3.8519	0.4436		0.6369	0.0469	0.1598	0.1564	0.0000
14		3.8465	0.4315		0.6300	0.0636	0.1553	0.1511	0.0000
15		3.8438	0.4228		0.6251	0.0755	0.1521	0.1473	0.0000
16		3.8424	0.4165		0.6215	0.0841	0.1498	0.1446	0.0000
17		3.8416	0.4121		0.6190	0.0902	0.1481	0.1427	0.0000
18	M-V Portfolio	3.8409	0.4008		0.6126	0.1057	0.1439	0.1377	0.0000

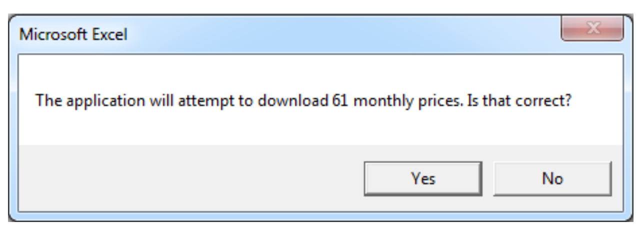
When you click "Run Program" the date selection dialog box pops up:

Figure 8. Date Selection Dialogue Box.



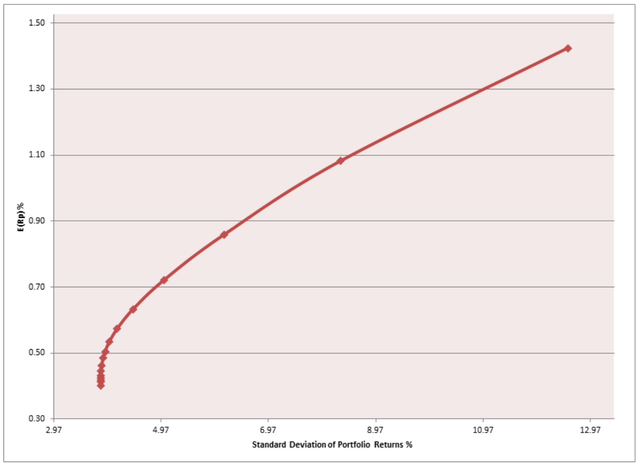
Use the mouse to select the desired start and end dates. There are several checks built into the dialog box such as the date cannot be in the future and the start date must be before the end date. Once again, the user is reminded that a minimum of 30 observations must be used. Once the dates are selected and the OK is clicked, a number of observations check dialog box is displayed:

Figure 9. Download Initiation Selection Box.



If the user clicks "No" they are returned to the date dialog box. After clicking "Yes", the user can watch the status bar as the program attempts to download the data from Yahoo and complete the optimization. Once completed, the "Graph" sheet is shown:

Figure 10. Markowitz Graph Output.



SUGGESTED CLASSROOM USE

UW-Whitewater runs two sections of undergraduate Investments each term with section sizes between 55 and 60 students. Lectures and exams cover the optimization process for two risky assets when r_f exists, but the undergraduate students rarely show a thorough understanding of the method. One way to increase their understanding is use real world data with more than 2 risky assets. With undergraduates, 5 risky assets is sufficient to determine if they can complete the optimization process and graph an efficient frontier. Instructions on how to download data from finance.yahoo.com are provided to students along with instructions on how to optimize a portfolio using Solver in Excel. Some textbooks contain this information but the undergraduate textbook currently in use at UW-Whitewater does not. The yahoo instructions are also available from the authors and instructions for Markowitz style optimization are available in many textbooks including Bodie, Kane and Marcus, *Investments* (McGraw-Hill Irwin), Chapter 7, Appendix A (pages 234-239 in the 9th edition) and Craig Holden's *Excel Modeling and Estimation in Investments* (Pearson Prentice Hall), Chapters 7 and 8 in the 3rd edition.

When class sizes were smaller (in the early 1990's Investment class sizes at Whitewater were in the 15-20 range), students were put into small groups and allowed to select their own securities. Grading of 6-10 assignments was not prohibitive and assignments could be returned in a few days. As class sizes grew, group sizes increased but class sizes reached the point where it was necessary to assign a single group of specific stocks for the entire class. These adjustments allowed the grading of 30-40 assignments in a reasonable timeframe but dramatically increased the amount of cross-group sharing. Requesting students behave ethically and increasing the penalty for cross-group sharing had little effect on the issue. It was clear that each group or student would need to be assigned a separate set of securities and that a more efficient method of grading had to be used; thus the development of the automated program.

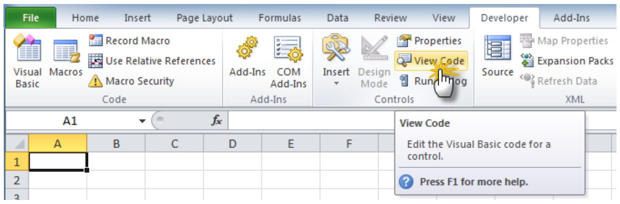
We use the program for several purposes. For the undergraduates, we randomly generate the 5 stock portfolios they use in their optimizations. If there are 40 groups, we randomly generate 40 5-stock portfolios. Since the program generates the graph for the random securities, it is possible to determine if the optimization produces a "good" graph before including that set of securities in the assignment. Students may select different points for their optimization than the automated program but the program significantly reduces the time needed to determine if the optimization was done correctly. For the graduates, they select their own 10 stock portfolios, but those assignments can still be easily graded since the program can download and optimize a 10 stock portfolio in about 5 seconds (assuming a reasonable Internet connection). We also use the program in-class to demonstrate to students that most efficient frontiers have a similar shape regardless of the time period, length of time period or number of securities in the portfolio. We find that simply discussing correlations and their relationship to the shape of the efficient frontier is never as effective as using real data. The students also get to see the entire process from hand calculating a two stock optimization, to using Excel to optimize a multi-stock portfolio, to a fully automated computation of an optimization.

Assigning every group their own set of stocks has significantly reduced the amount of cross-group sharing but free-riding within groups still exists. Use of student self-evaluations within the group helps with this issue but often students are more concerned they will have to work with the same student again in another class than they are with the ethics of giving another student a good evaluation when they were actually a free-rider.

CONCLUSION

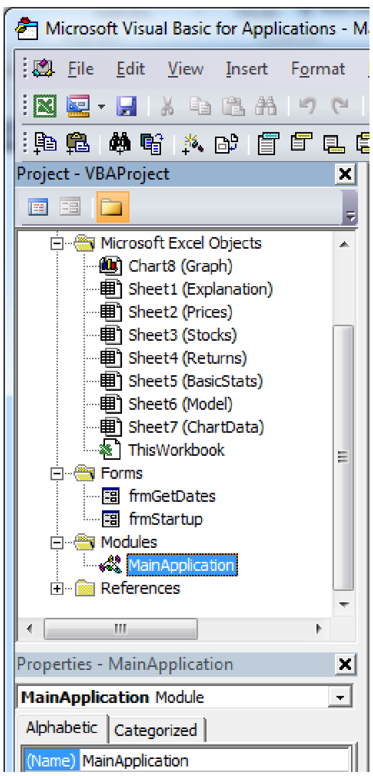
This paper described the "front-end" of an automated program for generating efficient frontiers using Microsoft Excel. The program can be used to reduce grading time while significantly curtailing cross-group sharing (cheating) when professors use assignments to improve student understanding of Markowitz (1959) optimizations. The program is freely available by contacting David Porter in the UW-Whitewater Department of Finance and Business Law. To minimize the distribution of the program to students, professors should use either their office phone or university email as evidence they are not a student. For those familiar with VBA, a PDF file is also available which explains the code thereby reducing the learning curve if changes are desirable to meet the individual needs of a specific course. To view the VBA code in the Excel file, use Alt+F11 or use the Developer tab. The Developer tab is not shown on the Excel toolbar by default but can be turned on by going to File | Options | Customize Ribbon and checking "Developer" under the "Main Tabs". The code can then be viewed by clicking on "View Code":

Figure 11. Developer Tab.



The main code can be found by double clicking on the Module "MainApplication":

Figure 12. Main Application Location.



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Demonstrating Retrieval of Financial Information in XBRL

Ameeta Jaiswal-Dale and Jianing (Jade) Fang

The Securities and Exchange Commission (SEC) has upgraded the existing system of Electronic Data Gathering, Analysis, and Retrieval (EDGAR), to Interactive Data Electronic Applications (IDEA) platform, using eXtensible Business Reporting Language (XBRL). In January 2009, the SEC issued its final mandate for XBRL adoption and the conversion target dates for all firms. This conversion enables users to retrieve listed companies' financial statement information at two levels, document and the data element, compared to the document level alone under the existing system of EDGAR. Beneficial to all users, this change is particularly important to resource-strapped entities such as small businesses, and universities. The result is user friendly for students researching information. With simply the industry standard tool of Microsoft Excel, without expensive proprietary XBRL modules, detailed information is now available, quickly, efficiently and at minimum cost. This paper explains the basic concept of XBRL and demonstrates the ease of the retrieval process in Microsoft Excel.

INTRODUCTION

On January 30, 2009, the Securities and Exchange Commission (SEC) chairperson, Mary Schapiro, announced the agency's final mandate for eXtensible Business Reporting Language (XBRL) adoption and the firm conversion target dates (SEC, January 2009). The mandate stated the largest domestic and foreign public companies that use U.S. Generally Accepted Accounting Principles (GAAP) to file their financial statements in XBRL format by June 15, 2009[1]: medium-sized filers by June 15, 2010, and the rest of the filers, either using U.S. GAAP or International Financial Reporting Standards (IFRS), by June 15, 2011 (Fang 2010).

The new rules are intended to make financial information easier for investors to analyze and to assist in automating regulatory filings and business information processing. The XBRL system lists information at both the document level, such as the entire set of financial statements for a given firm, and the data element level, such as individual accounts like inventory. In XBRL interactive data, or data tagged at the data element level, can function across multiple and/or different platforms or application programs. Thus, XBRL has the potential to increase the speed, accuracy and usability of financial disclosure, and eventually reducing costs for financial reporting as well as business transaction processing.

At the same time as the company files its financial statements or related registration statement with the commission in XBRL format, the SEC also requires a filer, for a minimum of twelve fiscal months, to post its financial statements in interactive data format on its corporate website. The new rules will not eliminate or alter existing filing requirements that financial statements, the accompanying reports and schedules be filed in the traditional format. The SEC believes that some investors and analysts may wish to use the traditional format to obtain an electronic or printed copy of the entire registration statement or report, either in addition to or instead of using interactive data.

In the traditional format, one painstakingly sifts through pages of statements and footnotes in order to extract the relevant data. Currently, whenever financial statement data are needed for fundamental analyses, users can download the related financial statements directly from the SEC's EDGAR system. For efficiency and/or accuracy, many users often turn to paid commercial data providers to obtain the necessary data. With Interactive Data Electronic Applications (IDEA), we will be able to retrieve these data directly from the SEC's website.

Beneficial to all users, this change is particularly important to resource strapped institutions, such as small businesses, students and universities. Using the industry standard tool of Excel, without expensive proprietary XBRL modules, these institutions can procure detailed information quickly, efficiently and at minimum cost.

The motivation for this paper is two-fold: to explain the basic concept of XBRL and to demonstrate the retrieval process in Excel in the classroom or in a workshop. Its contribution lies in drawing attention to the possibility of bypassing commercial data providers, for end users in financial services and at educational institutions.

Exhibit 1: SEC Web site

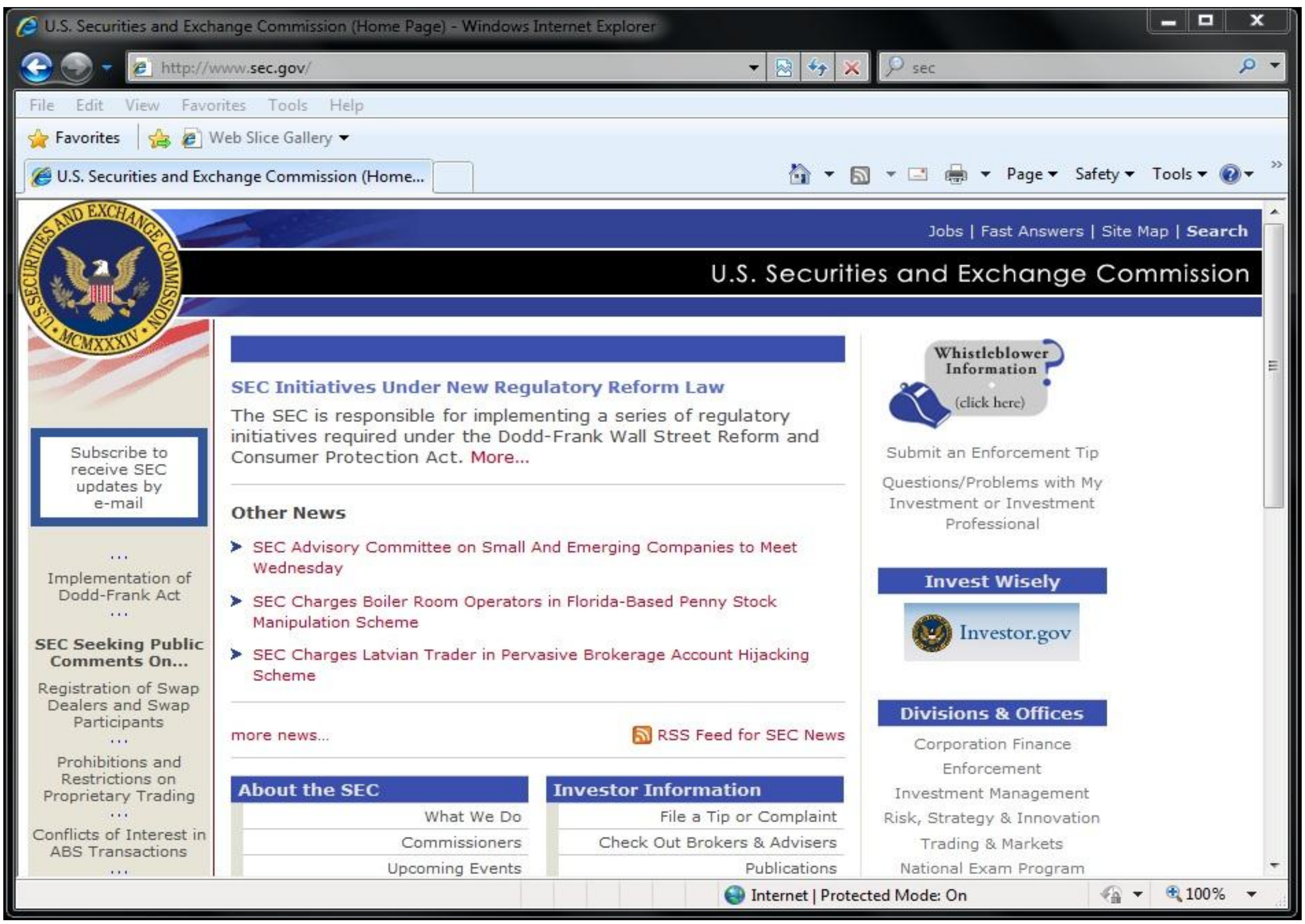
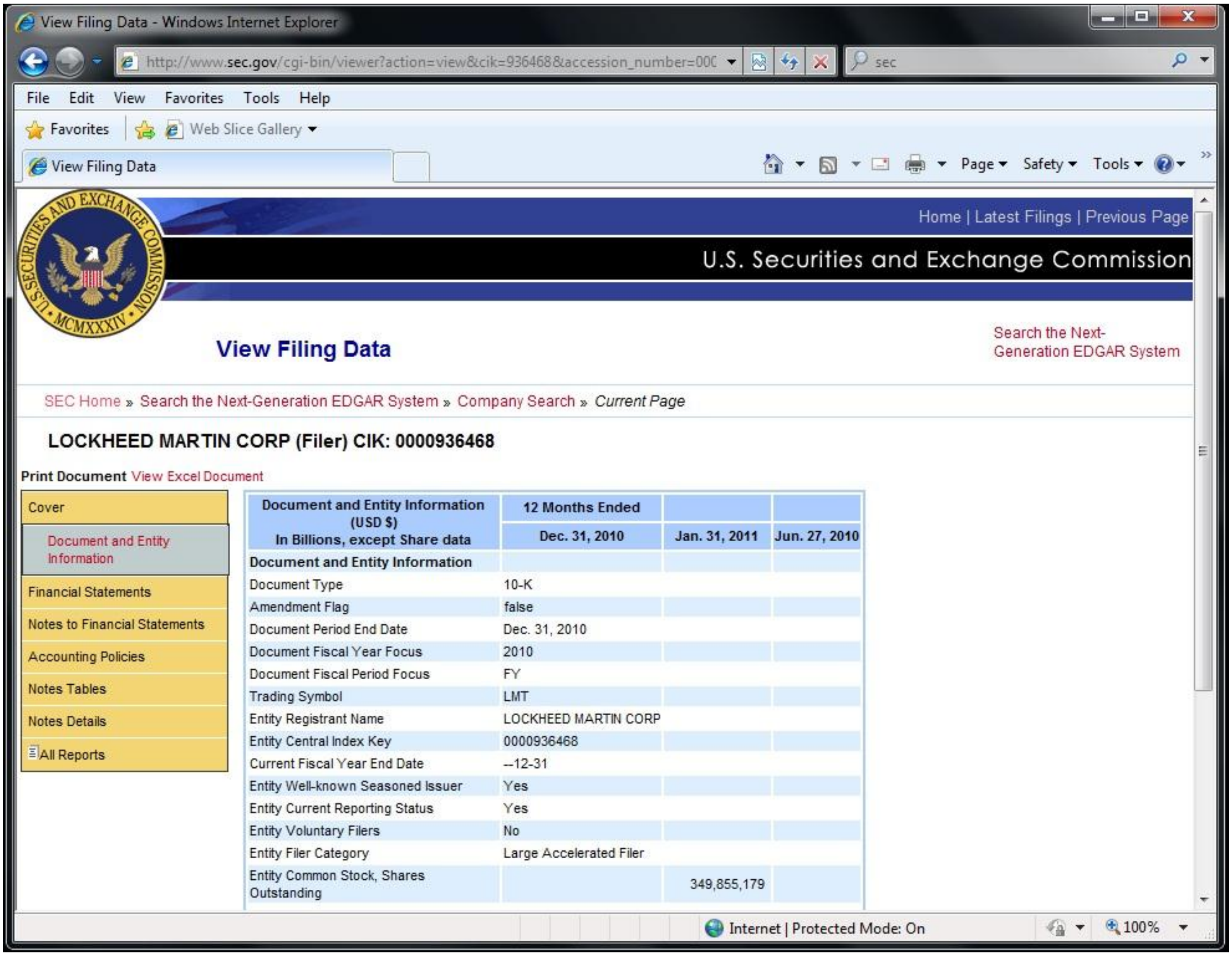


Exhibit 2: View Filing Data Window



The paper is organized in the following manner: section 2 explains some basic concepts of XBRL (Bizarro and Garcia 2010). Section 3 demonstrates XBRL data retrieval in Excel emphasizing how this would be useful for teaching and or employee training. Conclusion follows.

SOME BASIC XBRL CONCEPTS

At the beginning, listed companies filed their required reports with the SEC using paper. Later, during the 1990s, they filed electronic reports utilizing the EDGAR database. Now, aiming to provide investors and all other interested parties with faster and easier access to financial data, the SEC is upgrading the EDGAR system to another electronic platform: IDEA. In doing so, it is increasing capital market efficiency (Fang 2005 and Malkiel 2003) through information efficiency.

For illustrative purposes, assume that an investor wants to make a substantial investment. This investor chooses to perform a set of five-year profitability trend analyses for a pair of candidate companies to finalize his investment decision. Twenty-five years ago, when filing was paper-based, it would be necessary to first spend many hours in the library collecting information by hand from volumes of loose-leaf annual reports and then to perform all the necessary calculations. With the electronic system, only a fraction of that time is needed to collect similar data on either EDGAR or some other online data warehouse. This represents an amazing gain in efficiency. EDGAR is now upgraded to IDEA, a more efficient electronic system.

EDGAR provides users with electronic data at the document level. Companies file their financial reports in SEC prescribed forms, providing all the necessary data at the document level. This means that to perform your analysis, you first have to sift through hundreds of pages of financial statements. Then you need to manually re-enter this information in a spreadsheet or some kind of specialty software. Manual data entry is both time-consuming and potentially inaccurate.

IDEA, retaining some elements of EDGAR, uses XBRL to provide users with electronic data at the data element level (Phillips, Bahmanziari and Colvard, 2008).

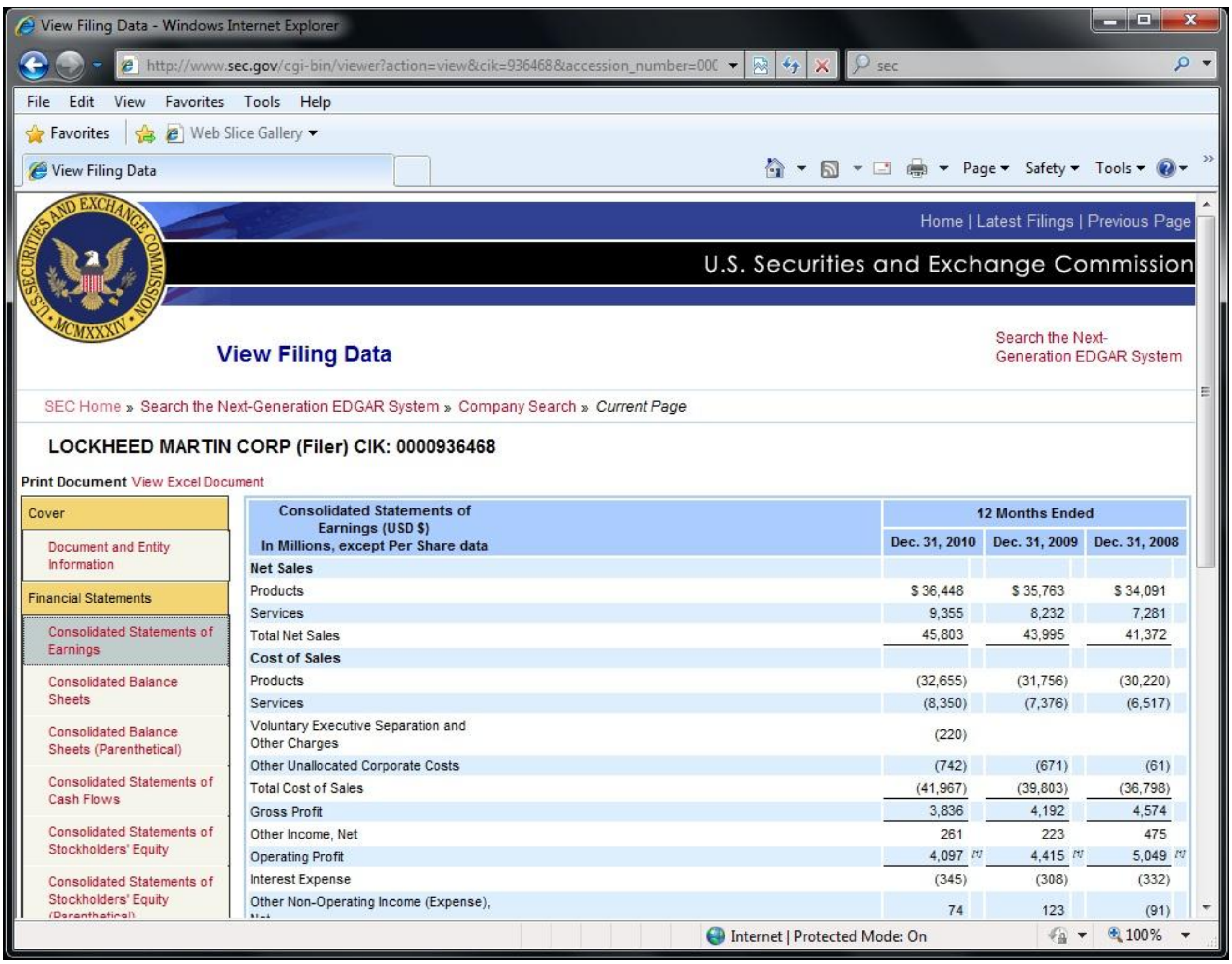
Financial reporting in XBRL provides accessibility, comparability and usability (Taylor and Dzuranin, 2010). Firms post on line a single XBRL instance document. This provides universal accessibility. The document contains industry specific approved taxonomies. The taxonomies serve as the means to tag data, footnotes and auditors reports. Tagging permits the association of a taxonomy element with a concept. This provides comparability across firms, search facilitated technology for comparison and transparency. Usability is enhanced via the standardized tags and elimination of transcribing data from one format to another, for example, from HTML to an analysis tool. Information is directly downloaded and then imported to a spreadsheet.

Now any application with XBRL processing capability can automatically import, cut, rearrange and present the necessary data in any shape or form according to user needs. As such, these applications are interactive. The elimination of the manual re-keying process not only improves data transmission efficiency, but also, and more importantly, data accuracy. IDEA has many advantages over EDGAR in terms of both speed and accuracy. Computers are much more efficient in searching, storing, and arranging information at the data element level, not just the document level (Fang 2009). In the next section, we will review examples of the use of information for financial analysis.

AN EXAMPLE FOR XBRL DATA RETRIEVAL: ASSET ALLOCATION FOR A STUDENT INVESTMENT FUND

The highlight of several business schools is their student investment funds. Student participation in the management of investment funds has been demonstrated to be a valuable technique for training individuals for the investment sector. The use of Microsoft Excel and XBRL makes this a cost effective proposition for most business schools. The following section demonstrates the convenience of XBRL in financial information retrieval using the ubiquitous tool of Microsoft Excel.

Exhibit 3: View Filing Data, 2008-2010



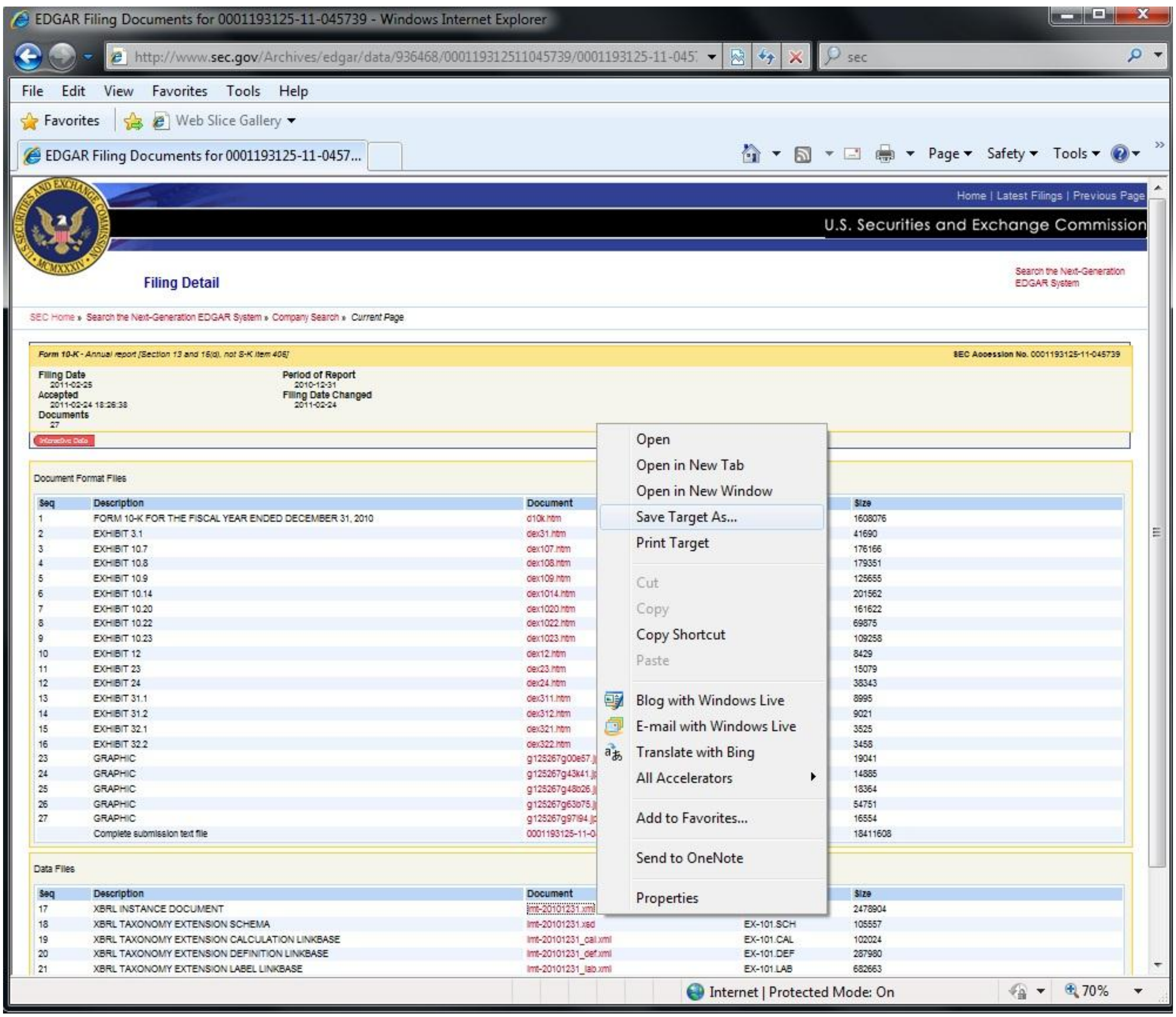
Consolidated Statements of Earnings (USD $) In Millions, except Per Share data	12 Months Ended		
	Dec. 31, 2010	Dec. 31, 2009	Dec. 31, 2008
Net Sales			
Products	$ 36,448	$ 35,763	$ 34,091
Services	9,355	8,232	7,281
Total Net Sales	45,803	43,995	41,372
Cost of Sales			
Products	(32,655)	(31,756)	(30,220)
Services	(8,350)	(7,376)	(6,517)
Voluntary Executive Separation and Other Charges	(220)		
Other Unallocated Corporate Costs	(742)	(671)	(61)
Total Cost of Sales	(41,967)	(39,803)	(36,798)
Gross Profit	3,836	4,192	4,574
Other Income, Net	261	223	475
Operating Profit	4,097	4,415	5,049
Interest Expense	(345)	(308)	(332)
Other Non-Operating Income (Expense), Net	74	123	(91)

This method is more efficient than the hand collection of data required to obtain data from 10-K reports filed with the SEC only a few years ago. This section provides a detailed classroom demonstration of the advantages of using this method for data retrieval from the SEC's IDEA platform into Microsoft Excel (2010).

A group of students, enrolled in an on-campus student-led investment fund class in finance, were assigned to pick the best military contractor to invest $2 million or 10% of the allowed investment fund. Using the capital asset pricing theory (Sharpe 1964), their professor has narrowed the candidates down to the final two companies: Lockheed Martin Corp. (LMT), and Honeywell International Inc. (HON). The students' task was to finalize the investment decision by undertaking a trend analysis comparing profit margin ratios over the last five years. In this section we demonstrate how the student can accomplish the task to retrieve the financial statements directly from the SEC's Next-Generation EDGAR system with Microsoft Excel. Our demonstration expands on that of Tribunella and Tribunella (March 2010) using Microsoft Excel 2007. The SEC has upgraded its IDEA system since then, so now some of their instructions are no longer functional. We extend their methods to the student-led investment fund using Microsoft Excel 2010.[2]

To begin, open a new Excel workbook. Immediately after opening a new Excel workbook, determine whether the "Developer" tab is displayed in the Ribbon. If not, left-click the File tab and then left-click on "Options" in the pane on the left-hand side This opens the "Excel Options" window. Choose "Customize Ribbon" in the left pane. In the right-hand column beneath "Customize the Ribbon", check the box next to "Developer."

Exhibit 4: Filing Window Detail



Left-click the “OK” button to confirm the selections and to return to the worksheet.

The next step is to download the necessary data. After opening an internet browser, go to the SEC website by typing in the web address (http://www.sec.gov) or by searching for “SEC” in an internet browser and clicking on the SEC link. From the SEC’s homepage, it is now possible to search for any one of the companies registered with the SEC by clicking on the “Search” button in the upper right corner (Exhibit 1). The SEC provides a few different ways to search company filings. For this sample analysis, the firm's financial statements will be located using the name search feature. First, type “Lockheed Martin Corp” into the “Company Name” field on the left search pane. Next, left-click on the “Find Companies” button. In the “Search Results” window, a list of all the documents that the company filed with the SEC is displayed chronologically, forty documents at a time, together with all the pertinent registrant information. A blue “Interactive Data” tag is used to identify all the filings in XBRL format.

By scrolling down the list, one can find the 2010 10-K report filed on 2011-02-25 with the SEC. Left-click on the “Interactive Data” tag, which will open the “View Filing Data” window (Exhibit 2). Left-click on the “Financial Statements” tag on the left pane and select “Consolidated Statements of Earnings” from the dropdown list, granting access to Lockheed Martin Corp’s consolidated income statements from 2008 to 2010 (Exhibit 3).[3] Left-click on the printer icon to print a hard copy and use it as a reference to check the accuracy of the XBRL data that will be downloaded later.

It is possible to save the information to a PC by first opening a new Microsoft Word document, switching back to the internet browser to copy the screen by holding down the “Ctrl” (“Fn” on some laptops) key and then press the “PrtSc” key on the keyboard. Then switch back to the Word document, right-click, then either select “Paste” or press “Ctrl” and “v” on the keyboard. These procedures will copy the screen shot to the Word document, which can then be saved for further use.[4] A second option exists for saving the reports in the Portable Document Format (PDF). Most internet browsers have the capability of either saving or “printing” a web page as a PDF. The major advantage with this second option is that instead of simply saving an image of the report, it is possible to save a searchable text document, significantly reducing time spent searching for relevant information later on in this process

Next, download the financial statement data in XBRL format. Switch back to the internet browser, navigate back to the document list window and scroll down to the same 10-K report but left-click on the “Documents” button this time. A “Filing Detail” window will open. Scroll down to find the file with the description “XBRL INSTANCE DOCUMENT” marked “Seq. 17” for this document. Right-click the file link named “lmt-20101231.xml” and choose "Save Target As …" see Exhibit 4. We recommend saving this file in the “XBRL” subdirectory that was created earlier.

The next step is to import the XBRL data needed for analysis to an Excel worksheet. Left-click on the Excel tab on the task bar at the bottom of the window. Open the instance document by left-clicking on the File tab, then click “Open”. Next, navigate to the “XBRL” subdirectory and select the “lmt-20101231.xml” file. After pressing the “Open” button, Excel can recognize that the file is in XML format.[5] This will provide an “Open XML” dialog box. Select the “Use the XML Source task pane” option and press the OK button. A warning box will appear, reporting that “…Excel will create a schema based on the XML source data.” Press the OK button to proceed. An “XML Source” pane will open on the right side of the worksheet. In order to map the data quickly and accurately, we recommend that displaying the associated value of each XML data element. Left-click the “Options” button at the bottom of the “XML Source” pane and check all the options.

The next step is to map the XBRL data elements equivalent to “Net Earnings” on the Excel worksheet. The problem is that XBRL is still in its infancy and all the XBRL taxonomies (XBRL financial statement filing standards for various accounting standards and/or industries, e.g., IFRS vs. US GAAP, general industry vs. insurance industry) provide many different labels to tag the same or similar data elements (Capozzoli and Farewell 2010). This presents a formidable challenge for the users—especially an XBRL novice. Looking for the correct tag from a list with hundreds of data elements for any new instance document is like searching for a tiny needle in a huge haystack because we do not know which label a given company used to tag a particular data element. This is why it was previously recommended to first either print out a hard copy of the Consolidated Statements of Earnings or to save a screen shot of it, and then to choose to display the data value in the “XML Source” pane.

After scrolling about 3/4 of the way down the list on the “XML Source” pane, a data element named “ns2:NetIncomeLoss” with a <value> of “3217000000” and “ContextRef (Duration_1_1_2008_to_12_31_2008)” will be visible. Refer to the hard copy or screen shot of the Consolidated Statements of Earnings for Lockheed Martin Corp. (Exhibit 5); it can be verified that the two amounts match each other. It is also possible to learn that the company uses the label of “ns2:NetIncomeLoss” to tag its “Net Earnings”.

Next, click on <value> and drag it to either cell (A1) on worksheet or to any other field appropriate for mapping the “Net Earnings” or XBRL data elements labeled “ns2:NetIncomeLoss.” Left-click on the “Developer” tab in the Excel Ribbon on top of the window and choose “Import”. Excel will display an “Import XML” window. After this, navigate to the “XBRL” subdirectory to select the same “lmt-20101231.xml” file. After pressing the “Import” button at the bottom of the window, Excel will import and place Lockheed Martin’s “Net Earnings” for all the reporting periods (2008, 2009 and 2010) below the XBRL data element of "ns2:NetIncomeLoss". We recommend labeling the data with the corresponding year by referring to the hard copy or screen shot of the Consolidated Statements of Earnings (Exhibit 5). Following the same procedure, scroll down the XML Source pane, about 4/5 of the way down the list, to find the "Total Net Sales" or the XBRL equivalent data elements of "ns2:SalesRevenueNet" and map it to cell "D1" on the Excel worksheet. Following the procedures described above, repeat the same process for the "Total Net Sales" data for the last three years. Right-click the worksheet tab "Sheet 1" at the bottom of the window and rename it as "LMT_Imp" and "Sheet 2" as "Report". Save the workbook in the "XBRL" subdirectory as "XBRL_Fin". Later, we will use the "LMT_Imp" worksheet as a template for importing all the necessary XBRL data and the "Report" worksheet for the analyses and reports. On the "Report" worksheet, choose a section for calculating the five-year Profit Margin Ratio change and trend analysis for Lockheed Martin Corp. The next step is to copy all the necessary data from your "LMT_Imp" worksheet to your "Report" worksheet. You should also setup all the formulas needed for the Profit Margin Ratio change trend analysis. For example, the formula in cell F6 is "=F3/F4," and in cell F8 is "=(F6-E6)/E6". You only need to enter each of these formulas once then copy and paste it to the other years, see Exhibit 6.

Following the same procedure, you can download LMT's financial statement data for 2009 and for many years to come. Now that you have setup the XBRL mapping and Report worksheet, you need a minute or two to retrieve the data without any "input errors"—a net gain in both efficiency and accuracy.

Exhibit 5: Mapping and Importing XBRL Data

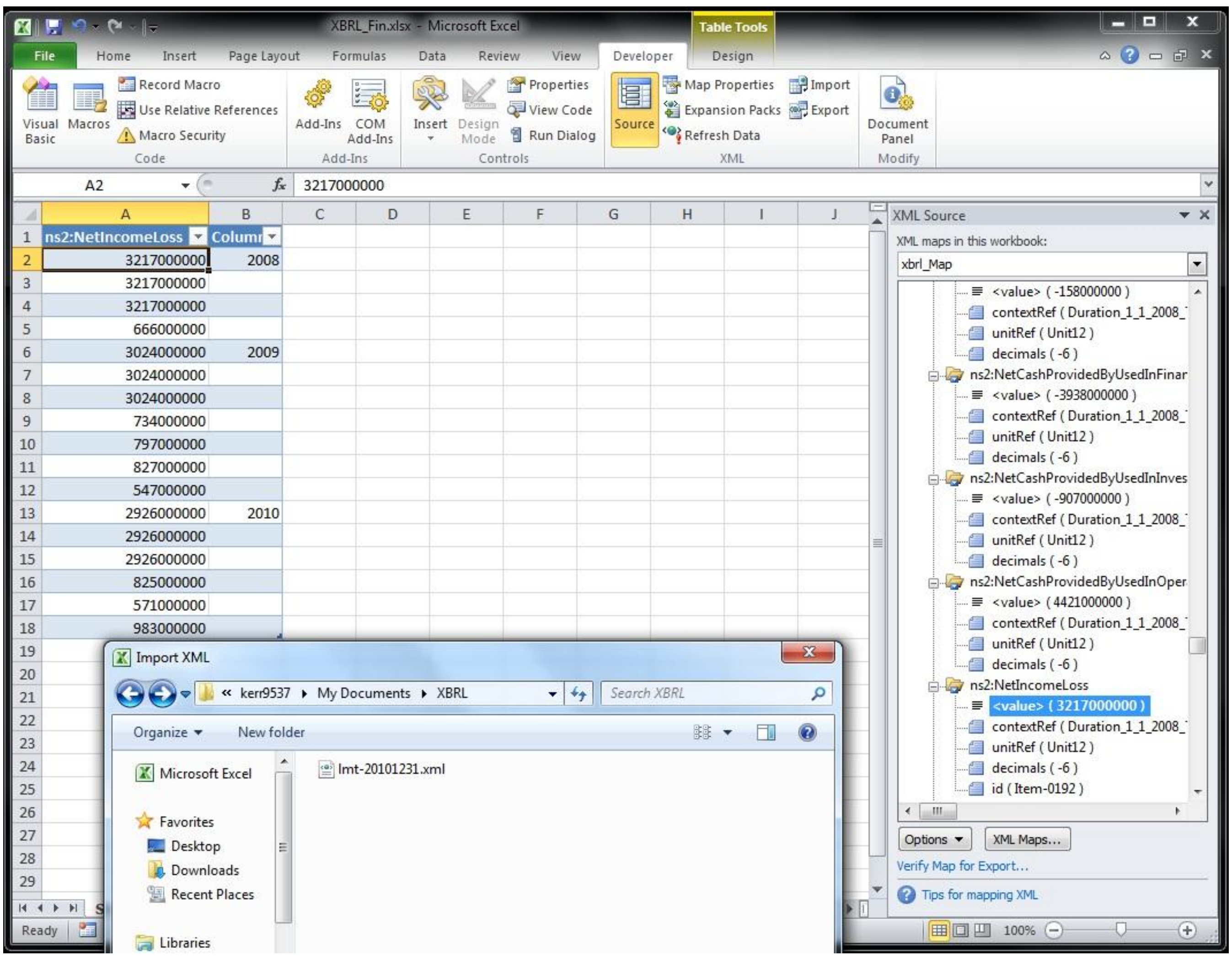
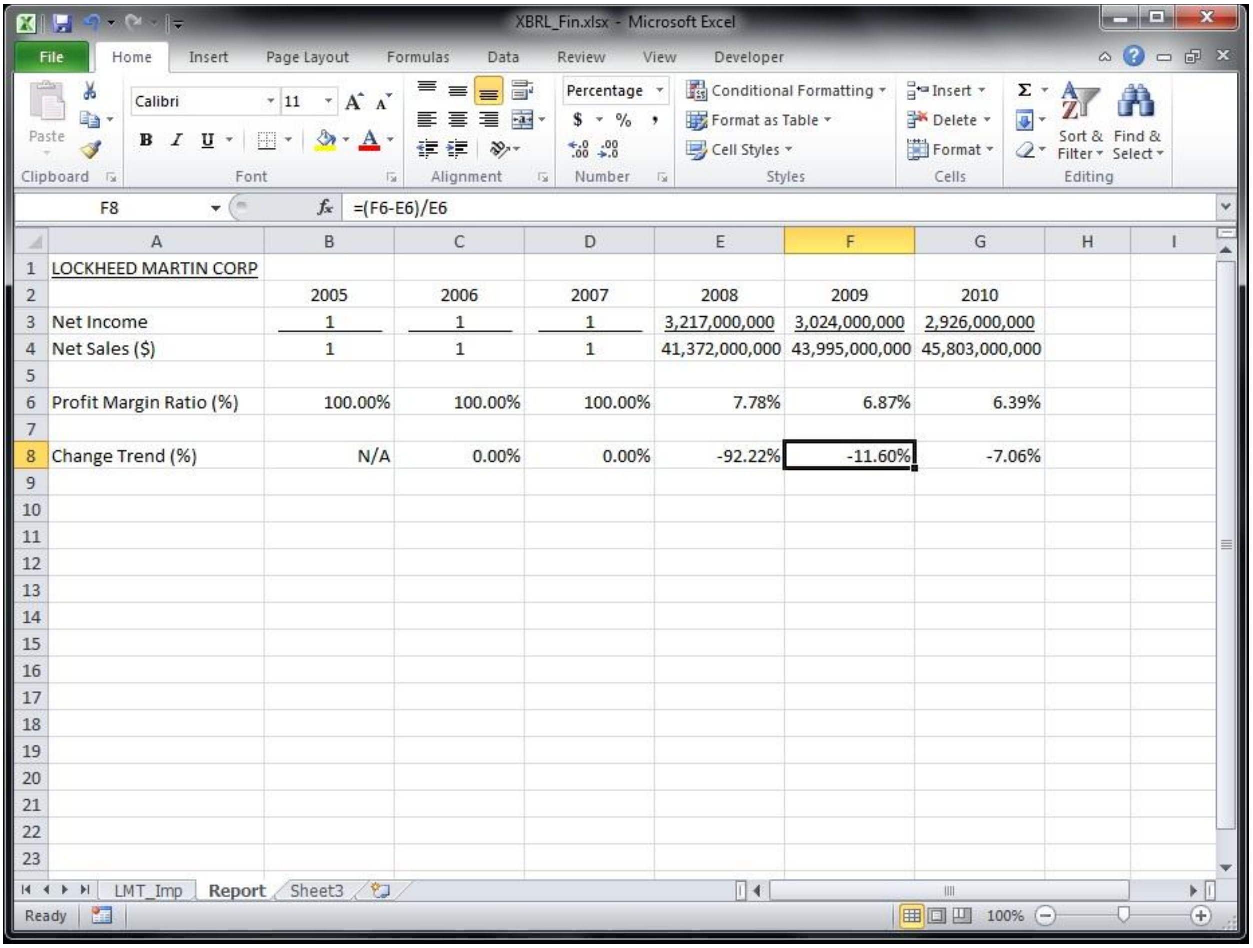


Exhibit 6: The Report Worksheet



	A	B	C	D	E	F	G
1	LOCKHEED MARTIN CORP						
2		2005	2006	2007	2008	2009	2010
3	Net Income	1	1	1	3,217,000,000	3,024,000,000	2,926,000,000
4	Net Sales ($)	1	1	1	41,372,000,000	43,995,000,000	45,803,000,000
5							
6	Profit Margin Ratio (%)	100.00%	100.00%	100.00%	7.78%	6.87%	6.39%
7							
8	Change Trend (%)	N/A	0.00%	0.00%	-92.22%	-11.60%	-7.06%

To complete this demonstration we wish to indicate that, with the exception of the companies that took part in the SEC's voluntary XBRL filing program, most have chosen to file their financial reports in XBRL format for just a year or two. It is possible to continue to download financial statements at the document level from EDGAR by following similar procedures to search for the target company (LMT for this example). After navigating to the "Filing Detail" window (Exhibit 4), click on the "d10k.htm" under "Document" on the top pane. EDGAR will provide the full 10-K report. It will be necessary to manually sift through the report to find the data needed for your analysis and enter the amounts in the "Report" worksheet. Hand-collection of data is much slower than the XBRL import, and more importantly, much more susceptible to "input errors". However, a search for information online or in a saved PDF may still improve search accuracy and efficiency. These formats maintain the integrity of the reports at the document level. By using the search function, (usually "Ctrl-f"), one can instantly locate every occurrence of a given word or phrase in a document. This method is still subject to human error; beware of misspellings.

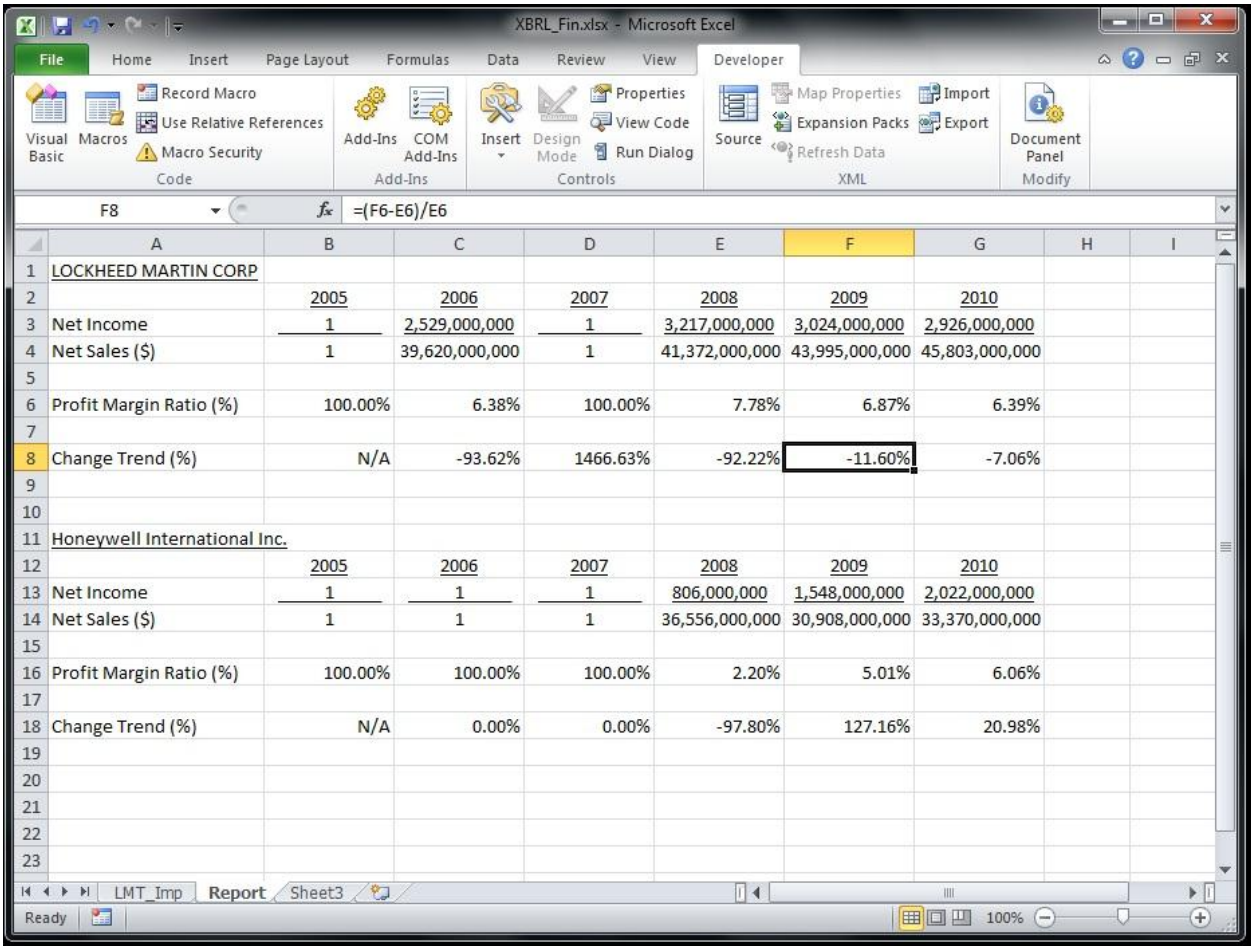
Finally, it is important to follow similar procedures to setup a separate Excel workbook as an XML import template for Honeywell International Inc. The finished report should look like the final report in Exhibit 7 ("1" is used as a placeholder for missing data).

CONCLUSION

Over the past two years, the SEC has made considerable progress in building a functional IDEA system. It has successfully implemented the Next-Generation EDGAR System, which provides its users with many convenient search options such as multiple links and RSS (really simple syndication) feeds. This new system also seamlessly combined the old document level EDGAR system with the new data element level IDEA system. Thus, a user can retrieve a company's financial reports in XBRL format as well as in electronic document format.

In this article, we have explained related concepts and demonstrated detailed steps describing how to directly retrieve XBRL data from reports filed with the SEC and import those data into Excel. To be better prepared to handle

Exhibit 7: The Final Report



	A	B	C	D	E	F	G
1	LOCKHEED MARTIN CORP						
2		2005	2006	2007	2008	2009	2010
3	Net Income	1	2,529,000,000	1	3,217,000,000	3,024,000,000	2,926,000,000
4	Net Sales ($)	1	39,620,000,000	1	41,372,000,000	43,995,000,000	45,803,000,000
5							
6	Profit Margin Ratio (%)	100.00%	6.38%	100.00%	7.78%	6.87%	6.39%
7							
8	Change Trend (%)	N/A	-93.62%	1466.63%	-92.22%	-11.60%	-7.06%
9							
10							
11	Honeywell International Inc.						
12		2005	2006	2007	2008	2009	2010
13	Net Income	1	1	1	806,000,000	1,548,000,000	2,022,000,000
14	Net Sales ($)	1	1	1	36,556,000,000	30,908,000,000	33,370,000,000
15							
16	Profit Margin Ratio (%)	100.00%	100.00%	100.00%	2.20%	5.01%	6.06%
17							
18	Change Trend (%)	N/A	0.00%	0.00%	-97.80%	127.16%	20.98%

this challenge in the near future, business schools and institutions with modest resources will still be able to train their faculty, students and/or staff with XBRL theories and skills. By now, most of the big software houses have developed their proprietary XBRL modules to work with their Enterprise Resource Planning systems for filing financial reports with the SEC as well as retrieving IDEA data for financial analyses. However, Microsoft Excel remains the most cost effective and accessible XML-capable software for most users of more modest resources.

ENDNOTES

1. There are approximately 500 such companies with worldwide common equity float above $5 billion at the end of the second fiscal quarter of their most recently completed fiscal year.

2. Everyone has different preferences and skill levels when using Excel and internet browsers. The authors are average users; the following procedures reflect their own preferences and abilities to get the job done using Microsoft Excel 2010.

3. In Exhibit 5, the accounting line items are interactive. Each line item has a filing code. For example, left-click on "net earnings" in Exhibit 5. This opens a window with three items – definition, references and details. Left-click on the details tab. This will show you "name:us-gaap_net income loss". This filing name is to be noted. It is used to identify net earnings when you are importing data.

4. We would like to take this opportunity to promote a good habit to help organize computer files in the most logical way for easy access. The authors created an "XBRL" subdirectory on a PC's "C" drive to store all the files related to XBRL research projects. This Word document was saved in that subdirectory as "LMT_HON,"a mnemonic file name. When working on multiple research projects concurrently, utilizing this filing system may be helpful in the retrieval of files at a moment's notice.

5. XBRL is one of the many dialects of XML.

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Demonstrating Portfolio Frontier Formation and the Capital Allocation Line Using Interactive Graphing

Pamela LaBorde and David Rystrom

This paper shows how to use interactive graphing via scroll bars to create a two-security portfolio frontier and the Capital Allocation Line. The frontier is enhanced via the use of spinners to control the correlation coefficient and the weight of risky assets as well as data labels that track the weight of the stocks along the frontier as a scroll bar is incremented. The VLOOKUP function is incorporated to lookup the calculated expected return and standard deviation of the risky portfolio at a stock weight controlled by a spinner. The purpose of this paper is to aid instructors in explaining the concepts of investment opportunity sets or frontiers, including the efficient and inefficient portions and riskless lending and borrowing to students of investing.

INTRODUCTION

This paper presents an Excel (Version 2010) spreadsheet[1] that interactively generates the portfolio frontier and the Capital Allocation Line (CAL) through the use of graphing, scroll bars and spinners and the VLOOKUP function. This technique allows the instructor to demonstrate: (1) the effect of changing the weight of securities in the portfolio; (2) the effect of the correlation coefficient on the position and shape of the frontier; and (3) the impact of incorporating a riskless asset upon the frontier. We have found this to be an excellent method of teaching these concepts without becoming mired in calculations and tediously redrawing the graphs by hand, and more effective than showing students static graphs from textbooks.

The paper proceeds as follows. The first section reviews the basic idea of portfolio theory and the "traditional" method of explanation and demonstration. The second section explains the process of creating the graph with the frontier of two risky assets, then adding the data labels and the spinner that controls the correlation coefficient. The final section incorporates a riskless asset and generates the CAL via scroll bars that control the borrowing and lending weights and includes a spinner to control the allocation of risky funds between two risky assets.

TRADITIONAL METHOD AND PRELIMINARY DISCUSSION

The two-security portfolio is the starting point for discussion of the frontier and the portfolio choice problem. The Expected Return (ER) of a two-security portfolio is:

$$\text{Expected Return}_{\text{portfolio}} = (w_1 \times ER_1) + (w_2 \times ER_2) \qquad (1)$$

and the equation for the Standard Deviation of the two-security portfolio is:

$$SD_{\text{portfolio}} = \sqrt{w_1^2 SD_1^2 + w_2^2 SD_2^2 + 2 w_1 w_2 SD_1 SD_2 \rho_{1,2}} \qquad (2)$$

where:

w_1 = weight in security 1,
w_2 = weight in security 2,
ER_1 = expected return (ER) for security 1,
ER_2 = expected return (ER) for security 2,
SD_1 = standard deviation (SD) for security 1,
SD_2 = standard deviation (SD) for security 2, and
$\rho_{1,2}$ = correlation coefficient between securities 1 and 2.

Traditionally, a textbook (*e.g.*, Hearth and Zaima [2006] or Bodie, Kane, and Marcus [2005]) will present these equations, enter some typical values and illustrate the calculations. Then the weights and correlation coefficient are varied to show the investment opportunity set and to explore the effect of different values upon the shape of the frontier. In classroom presentations, this may involve tedious calculations via the board or, alternatively, presentations via PowerPoint or overhead projections in which the results are already pre-drawn; these approaches may not leave the student with a clear understating of how the various points on the frontier are generated. Our interactive approach can be used to show immediately and clearly how changing inputs impact the position and shape of the frontier.

THE INTERACTIVE SPREADSHEET

Inputs

We'll begin with an empty spreadsheet tab. The inputs needed for the basic portion of the spreadsheet are the expected return (ER) and standard deviation (SD) for each of the two securities (STOCK1 and STOCK2) and the correlation coefficient ($\rho_{1,2}$) for the two securities. For later use we also include the return on a riskless asset. These data are all entered by the instructor--no calculations are involved. The basic inputs for this example are shown in Table 1. Input the values shown in Table 1 into the spreadsheet.

Table 1. Basic Inputs.

	A	B	C	D	E
1		Stock1	Stock2	Riskless Asset	Correlation Coefficient
2	Expected Return (ER)	22.20%	13.80%	3%	-0.46
3	Standard Deviation (SD)	12.11%	20.72%	0%	

Next, add the possible weights of STOCK1 in cells B7:B107 and labels as shown in B5:D6 as shown in Table 2A and 2B.

Table 2A. Weight, SD and ER for Risky Portfolio (Values).

	B	C	D
5		Risky Portfolio	
6	Weight in Stock1	SD	ER
7	0%	20.72%	13.80%
8	1%	20.46%	13.88%
9	2%	20.20%	13.97%
...			
105	98%	11.68%	22.03%
106	99%	11.90%	22.12%
107	100%	12.11%	22.20%

Table 2B. Weight, SD and ER for Risky Portfolio (Formulas)

	B	C	D
5		Risky Portfolio	
6	="Weight in "&B1	SD	ER
7	0	=SQRT((B7^2)*($B$3^2)+((1-B7)^2*($C$3^2)+(2*B7*(1-B7)*$B$3*$C$3*$E$2)))	=(B7*$B$2)+((1-B7)*($C$2))
8	0.01	=SQRT((B8^2)*($B$3^2)+((1-B8)^2*($C$3^2)+(2*B8*(1-B8)*$B$3*$C$3*$E$2)))	=(B8*$B$2)+((1-B8)*($C$2))
9	0.02	=SQRT((B9^2)*($B$3^2)+((1-B9)^2*($C$3^2)+(2*B9*(1-B9)*$B$3*$C$3*$E$2)))	=(B9*$B$2)+((1-B9)*($C$2))
...			
104	0.97	=SQRT((B104^2)*($B$3^2)+((1-B104)^2*($C$3^2)+(2*B104*(1-B104)*$B$3*$C$3*$E$2)))	=(B104*$B$2)+((1-B104)*($C$2))
105	0.98	=SQRT((B105^2)*($B$3^2)+((1-B105)^2*($C$3^2)+(2*B105*(1-B105)*$B$3*$C$3*$E$2)))	=(B105*$B$2)+((1-B105)*($C$2))
106	0.99	=SQRT((B106^2)*($B$3^2)+((1-B106)^2*($C$3^2)+(2*B106*(1-B106)*$B$3*$C$3*$E$2)))	=(B106*$B$2)+((1-B106)*($C$2))
107	1	=SQRT((B107^2)*($B$3^2)+((1-B107)^2*($C$3^2)+(2*B107*(1-B107)*$B$3*$C$3*$E$2)))	=(B107*$B$2)+((1-B107)*($C$2))

1. Enter the labels as shown in B5:D6.
2. Type "0%" in B7 and "1%" in B8. (NOTE ON QUOTATION MARKS: When a value or phrase is highlighted with quotation marks, do NOT input the quotation marks--the marks are merely to separate the information from the surrounding text. However, if quotation marks appear within a formula, the quotation marks should be included in the formula inputted.) Select the range B7:B8 with the mouse and use the "fill handle" (the small black square at the bottom right of the active cell or range) to drag the formulas to the range B9:B107.
3. Place the formula for Standard Deviation in C7:

=SQRT((B7^2)*($B$3^2)+((1-B7)^2)*($C$3^2)
+(2*B7*(1-B7)*$B$3*$C$3*$E$2))

C7 should now contain "0.2072," the SD of a portfolio composed of 0% in STOCK1 and 100% in STOCK2.

4. Place the formula for Expected Return in D7:

=(B7*$B$2)+((1-B7)*($C$2))

D7 should now contain the value "0.138," the ER of the portfolio. A useful exercise at this point is to ask the students to help generate the equations in Steps 3 and 4 by asking which cells should be referenced and if the cell references should be absolute or relative. This exercise offers a good opportunity to explain to students the difference between absolute and relative references and when each is applicable to a particular situation. Absolute references are used for the ER and SD of the two stocks as well as the correlation coefficient.

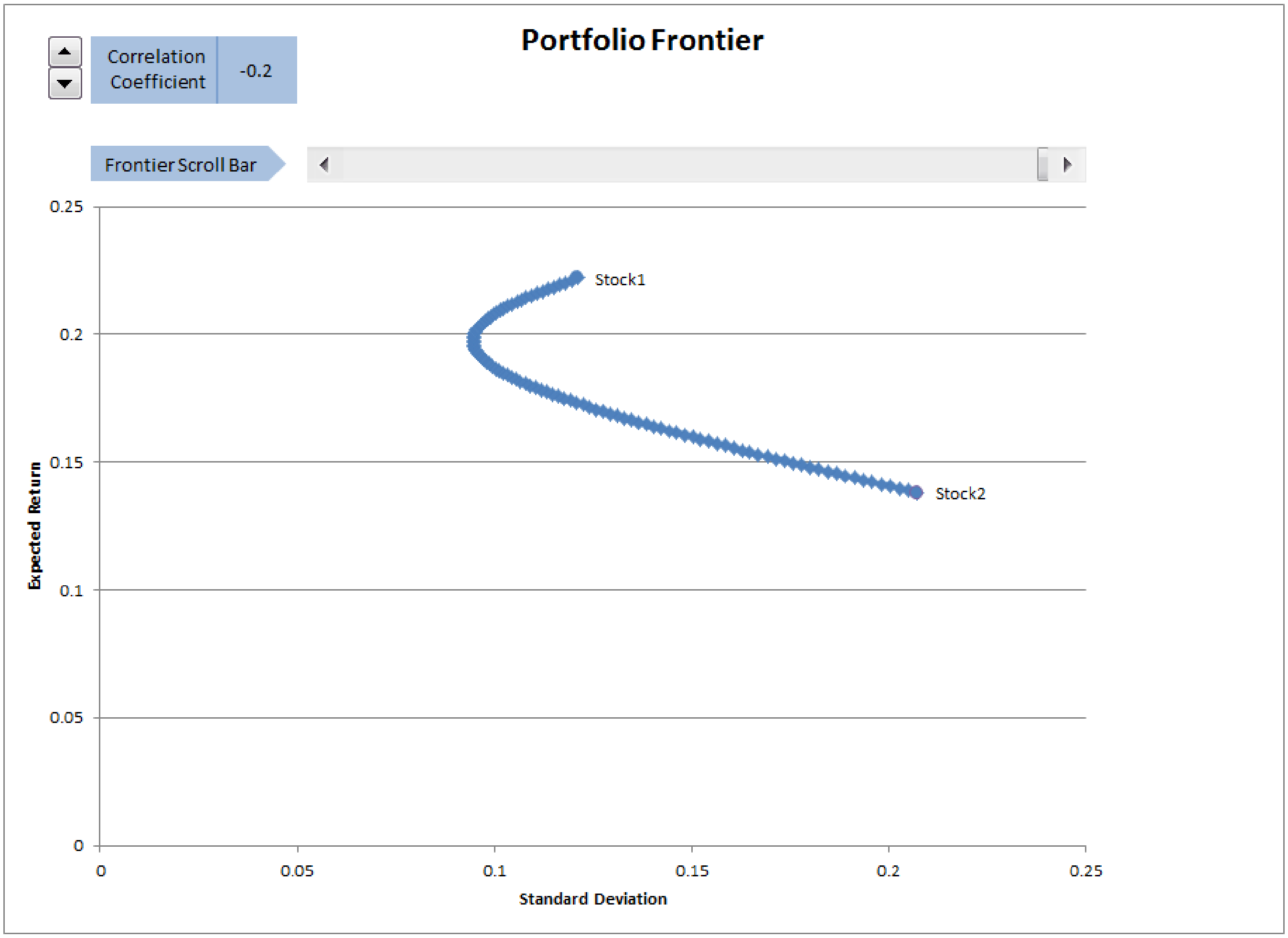
5. Select C7:D7 and use the fill handle to drag the formulas to C8:D107. Range C7:D107 now contains the expected return and standard deviation of the portfolio starting with 0% in STOCK1 and 100% in STOCK2 to 100% in STOCK1 and 0% in STOCK2 in increments of 1%.

The formulas for the values obtained in Table 2A are shown in Table 2B. It is important to note that the ONLY cells created during the lecture are B7:D107. The remainder of the spreadsheet discussed in the paper is calculated before the lecture. Now is a good time to perform several "housekeeping" functions. First, rename the worksheet tab to "Data" from "Sheet1." Secondly, save the spreadsheet.

Graphing the Frontier

While one could scroll down the list of values calculated in C7:D107 and discuss how risk and return changes at various weights of STOCK1 and STOCK2, showing the portfolio's ER and SD graphically helps students to intuitively grasp the issue of efficient vs. inefficient weightings. One could graph cells C7:D107 and show the entire frontier onscreen at once. However, there is some benefit to starting with an "empty" graph and using a scroll bar to draw the frontier. The chart we'll be creating for this section is shown in Figure 1. It is an XY Scatter Plot of the ER and SD of a portfolio composed of STOCK1 and STOCK2, with the weights changing in 1% increments via the "Frontier Scroll Bar" and a spinner controlling the value of the correlation coefficient between STOCK1 and STOCK2.

Figure 1. Graph with Frontier Scroll Bar and Correlation Coefficient Spinner



To create the graph and scroll bar:

1. Input the following formula in J7:

=IF($K$5/100>=$B7,C7,NA())

This formula checks the value of the counter (K5--which is currently empty but will be linked to a value created by the scroll bar), converts the value to a decimal, and compares this decimal value to the value stored in B7. B7 contains the weight of STOCK1. If the value of the counter is equal to or greater than the weight of STOCK1, the SD as calculated in C7 is entered into J7; otherwise the value NA() is entered. NA() returns the error value #N/A which means that no value is available. When a cell in a graphed data series contains the #N/A value, nothing appears on the graph for that point.

2. Using the fill handle, drag the formula in J7 to J8:J107. J7 should now contain the SD of the portfolio while J8:J107 should contain #N/A as K5 is currently empty. To test whether or not the formula is working, input a value of "3" into K5. The values of J8:J10 should change from #N/A to the SDs shown in C8:C10.

3. Copy the formulas in J7:J107 to K7:K107. K7:K10 should contain the ERs of the portfolio as shown in D8:D10 while K11:K107 should contain #N/A. Panel A of Table 3 shows the values while Panel B shows the formulas for a select range of this section of the spreadsheet.

4. Create an XY Scatter Plot similar to the one shown in Figure 1 by selecting J7:K107, choose the Insert ribbon, and selecting "Scatter" from the "Charts" section. Choose the "Scatter with only markers" chart option. Excel will automatically place J7:J107 as X and K7:K107 as Y. Four points should appear in the Plot Area of the graph as we've entered a test value of "3" in K5 and there are four XY data points in J7:K107. To avoid re-scaling of the X and Y axes as the scroll bar is scrolled, set the X axis scale to 25% by selecting the chart, then Chart Tools/Layout/Axes/Primary Horizontal Axis/More Primary Horizontal Axis Options and set the Axis Options maximum value to a "fixed" value of "0.25" and minimum value to "0." Do the same for the Y axis scale using the Primary Vertical Axis choice.

5. "Turn off" the legend and add axis labels and a chart title as shown in Figure 1 using the features under Chart Tools/Layout.

6. Finally, we prefer to place the chart in its own worksheet tab. To do this, right click in the "Chart Area"

(the area of the chart that does not include the actual plot of data points) of the newly-created chart, choose "Move Chart" and input "Frontier" in the "New Sheet" input box.

7. Resize the "Plot Area" by clicking the "Plot Area" and dragging the handle in the upper right corner so that the plot area is off-center with space above and to the right for the scroll bars and spinners.

Table 3. ER and SD For XY Scatter Plot (Frontier).
Panel A: Values

	J	K
5		3
6	Risky SD	Risky ER
7	20.72%	13.80%
8	20.46%	13.88%
9	20.20%	13.97%
10	19.93%	14.05%
11	#N/A	#N/A
12	#N/A	#N/A
…		
105	#N/A	#N/A
106	#N/A	#N/A
107	#N/A	#N/A

Panel B: Formulas

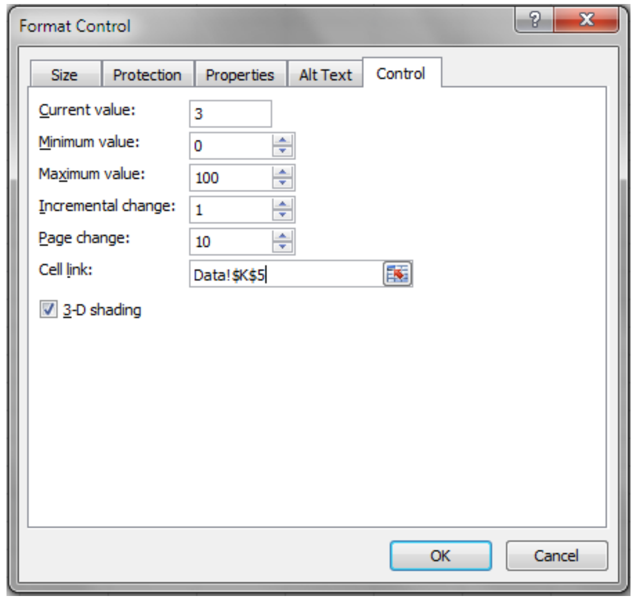
	J	K
5		3
6	Risky SD	Risky ER
7	=IF($K$5/100>=$B7,C7,NA())	=IF($K$5/100>=$B7,D7,NA())
8	=IF($K$5/100>=$B8,C8,NA())	=IF($K$5/100>=$B8,D8,NA())
9	=IF($K$5/100>=$B9,C9,NA())	=IF($K$5/100>=$B9,D9,NA())
10	=IF($K$5/100>=$B10,C10,NA())	=IF($K$5/100>=$B10,D10,NA())
11	=IF($K$5/100>=$B11,C11,NA())	=IF($K$5/100>=$B11,D11,NA())
12	=IF($K$5/100>=$B12,C12,NA())	=IF($K$5/100>=$B12,D12,NA())
…		
105	=IF($K$5/100>=$B105,C105,NA())	=IF($K$5/100>=$B105,D105,NA())
106	=IF($K$5/100>=$B106,C106,NA())	=IF($K$5/100>=$B106,D106,NA())
107	=IF($K$5/100>=$B107,C107,NA())	=IF($K$5/100>=$B107,D107,NA())

Adding the "Frontier Scroll Bar" to the graph requires accessing the Developer ribbon. If the Developer ribbon is not currently available, activate it by choosing File/Options/Customize Ribbon and check the box on the right side of the dialog box next to "Developer" under "Main Tabs." Click OK to return to your spreadsheet. To add the scroll bar:

1. Select the chart. On the Developer ribbon, choose Insert then choose "Scroll Bar (Form Control)" from the "Form Controls." Use the mouse to position the scroll bar on the spreadsheet. Right click the scroll bar and select "Format Control." Select the "Control" tab if it is not automatically chosen. The Control tab of the Format Control dialog box is shown in Figure 2.

The values to be entered are minimum value, maximum value, incremental change and cell link. The minimum value should be 0 (the lowest weight that can be placed into

Figure 2. Control Tab of the "Frontier Scroll Bar."



STOCK1), the maximum value should be 100 (the highest weight that can be placed into STOCK1) and the incremental value is the amount by which the counter will change when the scroll bar is scrolled with the arrows on either end--we use 1, or a 1% change in weight--you might wish to enter 5 or 10 so that the frontier is drawn more quickly. Cell Link refers to the sheet and cell in which you wish the counter to be placed. For this spreadsheet "Data" is the sheet name and $K$5 is the cell within the "Data" sheet.

2. Place your cursor in the "Cell Link" input box then click on cell K5 in your Data worksheet to link the scroll bar. Input the appropriate minimum, maximum and incremental values in the boxes, select OK and the scroll bar is ready for use. As the "Frontier Scroll Bar" is scrolled to the right, the counter increases in value in increments of 1 (or 1%) resulting in increasing values in K5; and the values in J8:K107 change from #N/A to the SD and ER of the portfolio. Since the frontier on the graph is created from cells J7:K107 (and not from C7:D107), as these cells change from #N/A to the values for SD and ER, the frontier is drawn.

3. Finally, add a label to the scroll bar by selecting the Frontier worksheet, selecting the chart, and choosing "Shapes" from the Insert ribbon. Choose "Pentagon" from the "Block Arrows" section. Place and size the arrow with the mouse. Right click the arrow and choose "Format Object" to adjust the colors, outlines, etc. Lastly, right click the arrow, select "Edit Text" and type "Frontier Scroll Bar" in the arrow.

For class presentation, the chart is already developed when demonstrating the spreadsheet to students. The chart is on a separate worksheet when first accessing the spreadsheet during lecture. The "Frontier Scroll Bar" is positioned at the minimum value when class begins. It is then possible to use the scroll bar on the frontier sheet to show the effect of changing the weights. As the scroll bar is incremented to the right, K5 increases in value (to a maximum of 100, or 100% weight in STOCK1).

Enhancing the Display: Use of Data Labels

To provide more information on the interactive chart, data labels can be used. For example, one could choose to display the weight of each stock based upon the current value of the scroll bar. It is also helpful to have the endpoints for the frontier labeled with the security names or ticker symbols such as those in B1 and C1.

To add the two endpoints:

1. Right click on the Chart Area and choose "Select Data." Choose "Add" in the "Legend Entries (Series)" dialog box. The cursor will be in the input box for "Series Name."
2. Select B1 (which contains the ticker symbol "STOCK1") from the Data worksheet. Next, place the cursor in the input box for "Series X values" and click on B3 in the Data worksheet. Data!B3 contains 12.11%. This is the SD for STOCK1.
3. Select the input box for "Series Y values" and delete the "{1}" (keep the "=" sign). Click on B2 from the Data worksheet. Data!B2 contains the ER for STOCK1, or 22.20%.
4. Choose "OK" and "OK" to close the dialog boxes and a new data point should appear on the graph. Move the slider for the Frontier Scroll Bar to the far left to make it easier to see the newly-created data point. Right click the data point and choose "Add Data Labels" then right click the data point again and choose "Format Data Labels." Under "Label Options" check "Series Name" for "Label Contains" and uncheck "Y Value." Close the dialog box and the data point should now be labeled "STOCK1."
5. If desired, you can change the marker options so that the data point blends with the frontier. To do so, right click the data point again and choose "Format Data Series." Select "Marker Options", set the "Marker Type" to "Built-in" and choose the circle. Change the color to blue, by using the "Marker Fill/Solid Fill" controls.
6. Repeat steps 1 through 5 for the STOCK2 data point, using "Data!C1" as the "Series Name," "Data!C3" as the "Series X values" and "Data!C2" as the "Series Y values."

Next, add the data labels that track the weight of STOCK1 and STOCK2 based upon the current value of the scroll bar as it is scrolled from the minimum to the maximum value. Although this section is a bit tedious, the results greatly enhance both the graph and student comprehension. Create this data label by inputting the following equation in L5 on the Data worksheet.

=CONCATENATE(K5,"% in ",B1,", ",
(1-K5/100)*100,"% in ",C1)

The CONCATENATE() function joins a group of characters and values into a single text string, which we can use as a data label. Assuming your scroll bar is set at the minimum value, L5 should contain "0% in STOCK1, 100% in STOCK2." As you move the scroll bar the value in K5 will change along with the data label in L5.

The data label in L5 will be used as a "Series Name" for a new series on the chart. To create this series we need to track whether or not a point is being displayed in range J7:K107. It is best to only display a label for the newest data point added by the scroll bar. Thus, if a point is not displayed, there should be no display of a data label. As newer data points are added via the scroll bar, this method provides the ability to "turn off" data labels on the points already graphed and "turn on" the data label for the newest point. This can be accomplished by activating a specific data point, graphing it and assigning to that specific point the data label just created in L5. The process requires nested AND() and ISNA() statements within an IF() statement.

L7:L107 will track whether a data label should be "on" or "off" for a data point.

1. Enter the following formula in L7 and using the fill handle drag the formula to L8:L107.

=IF(AND(ISNA(K7)=FALSE,ISNA(K8)=TRUE),K7, NA())

2. Set the scroll bar to a test value of 10 either by scrolling or typing "10" in K5. At this point, L17 should contain a value of "0.1464"--the ER of the portfolio composed of 10% in STOCK1 and 90% in STOCK2. All the remaining cells in L7:L107 should contain "#N/A." The ISNA() function returns a value of "TRUE" if a cell contains "#N/A" and "FALSE" if a cell does not contain "#N/A." The IF statement uses the following syntax: =IF (logical_test, [value_if_true], [value_if_false]). Do not include the brackets "[" or "]." Evaluate the formula for L17. L17 contains the following:

=IF(AND(ISNA(K17)= FALSE, ISNA(K18)= TRUE), K17, NA())

The logical_test [AND(ISNA(K17)= FALSE, ISNA(K18)= TRUE)] requires K17 to have a viable data point (i.e., is not equal to #N/A) and the next cell (K18) to not have a data point (i.e., is equal to #N/A). This makes K17 the newest data point created by the scroll bar. If both conditions are satisfied, the [value_if_true] activates and turns on the data point in L17 creating a point that can be graphed with the data label in L5. If either of the logical tests fail, the [value_if_false] activates and places #N/A in the cell. This situation occurred in L18, for instance, and all the remaining cells in the range L7:L107. Test the formulas in L7:L107 by typing in various values (weights) in K5 (between 0 and 100). The values in L7:L107 should remain #N/A except for the row that corresponds to the weight input in K5. Thus, if K5 contains the value "23," L30 should contain the value "0.15732" but the remaining values in L7:L107 should be #N/A. As the Frontier Scroll Bar is scrolled, L7:L107 will contain the ER for only the one data point we want to label. The SD for that data point is located in column J.

3. Add L7:L107 to the chart by right clicking on the chart, choosing "Select Data" and "Add." The "Series Name" is the data label in L5, or "=Data!L5", the "Series X Values" are "Data!J7:J107" and the "Series Y Values" are "Data!L7:L107". Select "OK" and "OK" to exit the dialog boxes. Assuming a value in K5 of "23," the chart should

contain a new data point at a SD of 0.15058 and an ER of 0.15732. It may be difficult to see the new data point as the point will be atop the last point generated by the Frontier Scroll Bar.

4. Right click this new data point, and choose "Add Data Labels." The data point should now have the label "0.15732." Right click the data point, choose "Format Data Labels" and in the "Label contains" section check the box for "Series Name" and uncheck the box for "Y value." Set the "Label Position" to "Left" and Close the dialog box. While moving the scroll bar, the data point should track the ER and SD for the portfolio given the current weight of STOCK1 as determined by the scroll bar. The data label should change to display the weights that determine that data point.

The result is a dynamic chart--the values will automatically adjust as changes are made in the basic inputs located in B1:C3. The interactive nature of the chart grabs the students' attention and the changing data labels showing the weights of STOCK1 and STOCK2 make it clear to the students exactly what is occurring as the frontier is created via the Frontier Scroll Bar.

Pedagogic Use of the Interactive Feature

Classroom discussion can utilize the interactive feature to discuss what portion of the frontier is efficient vs. inefficient. This can be shown by moving the scroll bar to show different points on the opportunity set. The students observe the greater ERs on the higher, efficient part of the frontier, as compared to the lower inefficient corresponding points for any SD. At the same time, the data label displays the weights that produce those combinations. Students can easily follow and understand the idea of dominant portfolios and the efficient part of the opportunity set from this demonstration. Additionally, students can observe what weights in STOCK1 and STOCK2 cause the frontier to switch from the efficient portion to the inefficient portion--the minimum standard deviation portfolio.

The instructor can also effectively use the interactive feature to demonstrate the impact of a change in the correlation coefficient upon the frontier. We'll add a spinner control to the chart that determines the value of the correlation coefficient in Data!E2. This should give the student a clear visual sense of the impact of the correlation coefficient on the shape of the frontier.

Spinner Control for Correlation Coefficient

Since the chart and the data are on two separate worksheets, we add a spinner control to the chart along with a text box that shows the current value of the correlation coefficient to the graph.

1. In the Frontier worksheet, choose "Insert" from the Developer ribbon and select "Spin Button (Form Control)." With the mouse, place and size the spin button in the upper left of the chart. Right click the spin button (or spinner) and select "Format Control." The minimum and maximum values need to represent the possible range for the correlation coefficient, or 1 to +1. Unfortunately, minimum value does not allow negative numbers nor is the incremental change allowed to be less than 1. Therefore, set the minimum value to "0," the maximum value to "200," and the incremental change value to "10." Finally, set the cell link to "Data!F2."

2. The spinner places a value ranging from 0 to 200 in Data!F2. However, correlation coefficients can only be in the range of -1 to +1. To convert the spinner range to that of the correlation coefficient, we can divide the spinner value by 100 (giving us a minimum value of 0 and a maximum value of 2). By subtracting 1 from the result, the range of the correlation coefficient, the desired range extending from a minimum value of -1 and a maximum value of +1, is created. Additionally, it is necessary to replace the original correlation coefficient value in cell E2 with the converted value generated by the spinner. To do so, on the Data worksheet enter the following equation in E2:

=F2/100-1

Set the scroll bar to its maximum position and note how the shape of the frontier changes as you adjust the spinner. To display the value of the correlation coefficient, we'll add a text box on the graph.

3. On the Frontier worksheet, select "Text Box" from the Insert ribbon. Using the mouse, draw a text box next to the spinner. Place your cursor in the formula bar (the input area to the right of the *f*x) and type "=Data!E2." The current value for the correlation coefficient should appear in the text box. Note that if you enter the formula into the textbox itself and not via the formula bar, the text box will display "=Data!E2" and not the actual value. Give the spinner a whirl to ensure the value in the text box adjusts with the spinner. You may wish to add a second text box with the text "Correlation Coefficient" as shown in Figure 1.

ADDING THE RISKLESS ASSET AND THE GENERATION OF THE CAPITAL ALLOCATION LINE

After the students are familiar with and understand the development of the frontier and the efficient/inefficient portions of the frontier, it is possible to introduce the idea of combining risky portfolios on the frontier with the riskless asset. These combinations are sometimes called the Capital Allocation Line (CAL)[2], the term used in this paper. It is useful to begin with a risky portfolio on the frontier that is not the tangency point for the CAL. This example uses the risky portfolio that is equally-weighted between STOCK1 and STOCK2. Typically an equally-weighted portfolio is relatively easily envisioned by students. This portfolio is denoted as risky portfolio E as shown in Figure 4.

Next, form a portfolio of 70% of our funds in portfolio E and 30% in the risk-free asset as another example. The use of dollar amounts can help clarify the weighting. As an example, begin with an investment of $100,000, and ask what ER and SD will result if $30,000 is invested in the riskless asset and $70,000 in portfolio E. If the riskless asset generates a return of 3% and risky portfolio E has an ER of 18% and a SD of 9.29% (assuming a correlation coefficient of -0.46 between STOCK1 and STOCK2), then the portfolio composed of 30% in the riskless asset and 70% in portfolio E will have an ER of 13.5% and a SD of 6.5% as demonstrated below:

$$ER_{E,\ risk\text{-}free} = (0.3)(0.03) + (0.7)(0.18) = 13.5\% \qquad (3)$$

$$SD_{E,\ risk\text{-}free} = [(0.3^2)(0.0^2) + (0.7^2)(0.0929^2) + 2\,(0.3)\,(0.7)\,(0)\,(0.0929)\,(0)]^{1/2}$$
$$= [0 + (0.7^2)(0.0929^2) + 0]^{1/2} = (0.7)(0.0929) = 6.5\% \qquad (4)$$

It is helpful to work this out "by hand" and discuss where this point would plot on the Frontier graph.

After illustrating these ideas with two different portfolios, the next step is to show students via the graph in Figure 4 how combining various weights (both positive and negative) of the riskless asset with risky portfolio E results in a line connecting the risk-free return and risky portfolio E. The next section shows how to add the Lending CAL and Borrowing CAL scroll bars to the graph.

Basic inputs for the CAL

The process for generating these scroll bars is similar to that used for the "Frontier Scroll Bar." To generate the Lending CAL, on the Data worksheet:

1. Enter the risk-free rate (3%) in D2 and the SD of the riskless asset (0%) in D3 if not already entered.
2. Create a section of the spreadsheet that contains the ER and SD for the risky portion of the portfolio--portfolio E in Figure 4. The SD and ER have already been calculated in columns C and D; it is merely a matter of having Excel find the values for the specific weight of STOCK1 and STOCK2 that create risky portfolio E.

Panel A of Table 4 shows the section of the spreadsheet used to extract the necessary data while Panel B contains the formulas. P4 contains the weight of STOCK1 for risky portfolio E and is a value determined by the user of the spreadsheet. For the example in Equations 3 and 4, P4 would have a value of 50%, or 0.50 as the risky portfolio E is comprised of equal weights in STOCK1 and STOCK2.

Table 4. Basic Inputs for CAL.
Panel A: Values

	P	Q	R	S
3	Weight STOCK1	Weight STOCK2	Risky ER	Risky SD
4	50%	50%	18.00%	9.29%

Panel B: Formulas

	P	Q	R	S
3	Weight STOCK1	Weight STOCK2	Risky ER	Risky SD
4	0.5	=1-P4	=VLOOKUP($P$4,$B$7:$D$107,3)	=VLOOKUP($P$4,$B$7:$D$107,2)

1. Add labels in P3:S3 as shown in Table 4 (Panel A).
2. Enter a value of "0.5" in P4. Q4 is calculated as (1 – P4) or 1 minus the weight of STOCK1. Enter "=1-P4" in Q4. Next, use the VLOOKUP formula to find the corresponding ER and SD for the risky portfolio comprised of P4 weight of STOCK1 and Q4 weight of STOCK2.
3. Enter the following formula in R4:
   =VLOOKUP($P$4,$B$7:$D$107,3)

The VLOOKUP (Vertical LOOKUP) formula uses the following convention: VLOOKUP (lookup_value, table_array, col_index_num). The lookup_value is the weight in P4. Excel will attempt to find that value in the table_array, or the range $B$7:$D$107. If it finds the value contained in P4 in the first column of $B$7:$D$107, it will return as an answer the value in the third (3) column, or col_index_num, of the corresponding row. R4 should contain the ER for a portfolio weighted 50% in STOCK1 and 50% in STOCK2, or "0.18."

4. Enter the following formula in S4:
   =VLOOKUP($P$4,$B$7:$D$107,2)

The formula performs the same function as the one in R4 except it returns the SD from the second column in the range of B7 to D107. Cells R4 and S4 now contain the ER and SD of risky portfolio E for generating the CAL. If you wish to evaluate the effect of different weights of STOCK1 and STOCK2 for the risky portfolio E, you only need to change the value in P4. Excel will look up the SD and ER of the new risky portfolio E and place the results in R4 and S4. Next, add a point for risky portfolio E to the graph.

5. From the Frontier worksheet, right click the Chart Area and select "Select Data" then "Add." Input "E" as the "Series Name," "Data!S4" as the "Series X values" and "Data!R4" as the "Series Y values." Select OK and OK to close the dialog boxes. While it may be difficult to see, a new data point is now on the graph--at the bottommost point where the CAL intersects the frontier. It is easier to find the new data point if the Frontier Scroll Bar is in the minimum position.
6. Right click this new point and select "Add Data Labels." The label "0.18" should appear after you close the dialog box. Right click the new point again, select "Format Data Labels," check the box by "Series Name," uncheck the "Y Value" box and check the "X Value" box. Also, change the "Label Position" to "Above." Select "Close" to exit the dialog box. The result is a data label with point E and the accompanying SD.
7. Finally, right click the point one final time, select "Format Data Series," choose the square "Built-in" Marker

Option and set the "Marker Fill" to a "Solid Fill" in a bright color.

Next, generate the ER and SD data for combined risky portfolio E and the risk-free asset, then generate another two columns that contain the data points to be graphed for the Lending CAL as shown in Panel A of Table 5. Range H7:I107 on the Data worksheet contains the information used to create the data points to be graphed as the "Lending CAL" scroll bar is scrolled in the same way J7:J107 generates the frontier as the Frontier Scroll Bar is scrolled. Initially H7:I107 contain the value #N/A, but the values change to the SD and ER as the Lending CAL scroll bar is scrolled. Panel B of Table 5 shows the formulas used to generate these data.

E7:E107 contains the weights for the risky portfolio E, while F7:G107 contain the SD and ER for the combined risky portfolio E and the riskless asset. The formulas for SD and ER in F7:G107 obtain the values for the risky portfolio portion with the weight combination established by the value in P4 and the SD (S4) and ER (R4) for that combination of risky assets.

Incorporate the information from Table 5 into your spreadsheet:

1. Enter the labels in E6:I6 into the Data worksheet.

2. Input the weights of "0%" and "1%" into E7 and E8, respectively, then highlight E7:E8 with the mouse and use the fill handle to drag the weights into E9:E107.

Table 5. Inputs for the Lending CAL Scroll Bar.
Panel A: Values

	E	F	G	H	I
6	Weight (Risky)	SD	ER	CAL Graph Inputs	
7	0%	0.00%	3.00%	0.00%	3.00%
8	1%	0.09%	3.15%	#N/A	#N/A
9	2%	0.19%	3.30%	#N/A	#N/A
…					
105	98%	9.10%	17.70%	#N/A	#N/A
106	99%	9.20%	17.85%	#N/A	#N/A
107	100%	9.29%	18.00%	#N/A	#N/A

3. Input the following equation into F7:

=E7*$S$4

This equation calculates the SD for the combination of risky portfolio E and the riskless asset.

4. Input the following equation into G7:

=((1-E7)*$D$2)+E7*$R$4

This equation calculates the ER for the combination of risky portfolio E and the riskless asset.

5. Select F7:G7 with the mouse and use the fill handle to drag the equations to F8:G107.

Just as the Frontier Scroll Bar incremented a counter that was placed in cell K5 the Lending CAL Scroll Bar needs a counter cell reference as well. We'll use N7 as the Lending CAL Scroll Bar's counter cell. This cell will reflect the value of the Lending CAL Scroll Bar and will change the weight of the investment in risky portfolio E. For now, simply type in a value of "10" in N7--this represents a 10% weight in risky portfolio E and, therefore, a 90% weight in the riskless asset. Later we will replace the "10" in N7 with a value generated by the Lending CAL Scroll Bar. First, we need to "turn on" the data points for the Lending CAL for the graph.

1. Input the following formula in H7:

=IF($N$7/100>=E7,F7,NA())

This formula checks the value of the counter (N7), converts it to a decimal, and compares it to the value in E7, which contains the weight of risky portfolio E. If the value of the counter is equal to or greater than the weight of risky portfolio E, the SD as calculated in F7 is entered in H7 (and subsequently will appear on the graph); otherwise the value NA() is entered (and will NOT appear on the graph).

2. The ER for the Lending CAL is determined in the same manner as the SD explained in the previous step. Input the following formula in I7:

=IF($N$7/100>=E7,G7,NA())

This formula will populate the ERs on the graph for the risky portfolio E and the riskless asset.

3. Select the range H7:I7 and drag the formulas to H8:I107. Since the counter cell N7 is set to a weight of "10," H7:I17 should contain the SD and ER from F7:G17, while range H18:I107 should contain #N/A.

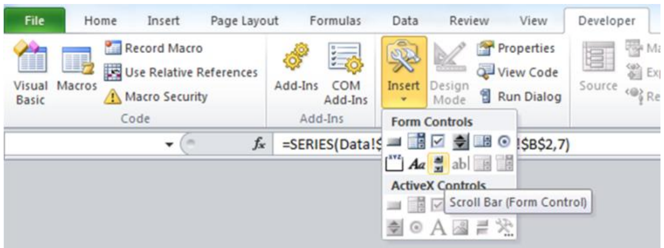
Table 5. Inputs for the Lending CAL Scroll Bar.
Panel B: Formulas

	E	F	G	H	I
6	Weight (Risky)	SD	ER	CAL Graph Inputs	
7	0	=E7*$S$4	=((1-E7)*$D$2)+E7*$R$4	=IF($N$7/100>=E7,F7,NA())	=IF($N$7/100>=E7,G7,NA())
8	0.01	=E8*$S$4	=((1-E8)*$D$2)+E8*$R$4	=IF($N$7/100>=E8,F8,NA())	=IF($N$7/100>=E8,G8,NA())
9	0.02	=E9*$S$4	=((1-E9)*$D$2)+E9*$R$4	=IF($N$7/100>=E9,F9,NA())	=IF($N$7/100>=E9,G9,NA())
…					
105	0.98	=E105*$S$4	=((1-E105)*$D$2)+E105*$R$4	=IF($N$7/100>=E105,F105,NA())	=IF($N$7/100>=E105,G105,NA())
106	0.99	=E106*$S$4	=((1-E106)*$D$2)+E106*$R$4	=IF($N$7/100>=E106,F106,NA())	=IF($N$7/100>=E106,G106,NA())
107	1	=E107*$S$4	=((1-E107)*$D$2)+E107*$R$4	=IF($N$7/100>=E107,F107,NA())	=IF($N$7/100>=E107,G107,NA())

Now it is time to add the Lending CAL Scroll Bar to the graph and attach the value of the scroll bar to the counter cell N7.

1. While in the Frontier worksheet, access the Developer ribbon, select "Insert" and choose "Scroll Bar (Form Control)." See Figure 3 for a screenshot of this procedure.

Figure 3. Insert Menu of Developer Ribbon.



With the mouse, position and size the scroll bar vertically on the right side of the plot area of the graph, as shown in Figure 4. Right click the scroll bar and select "Format Control."

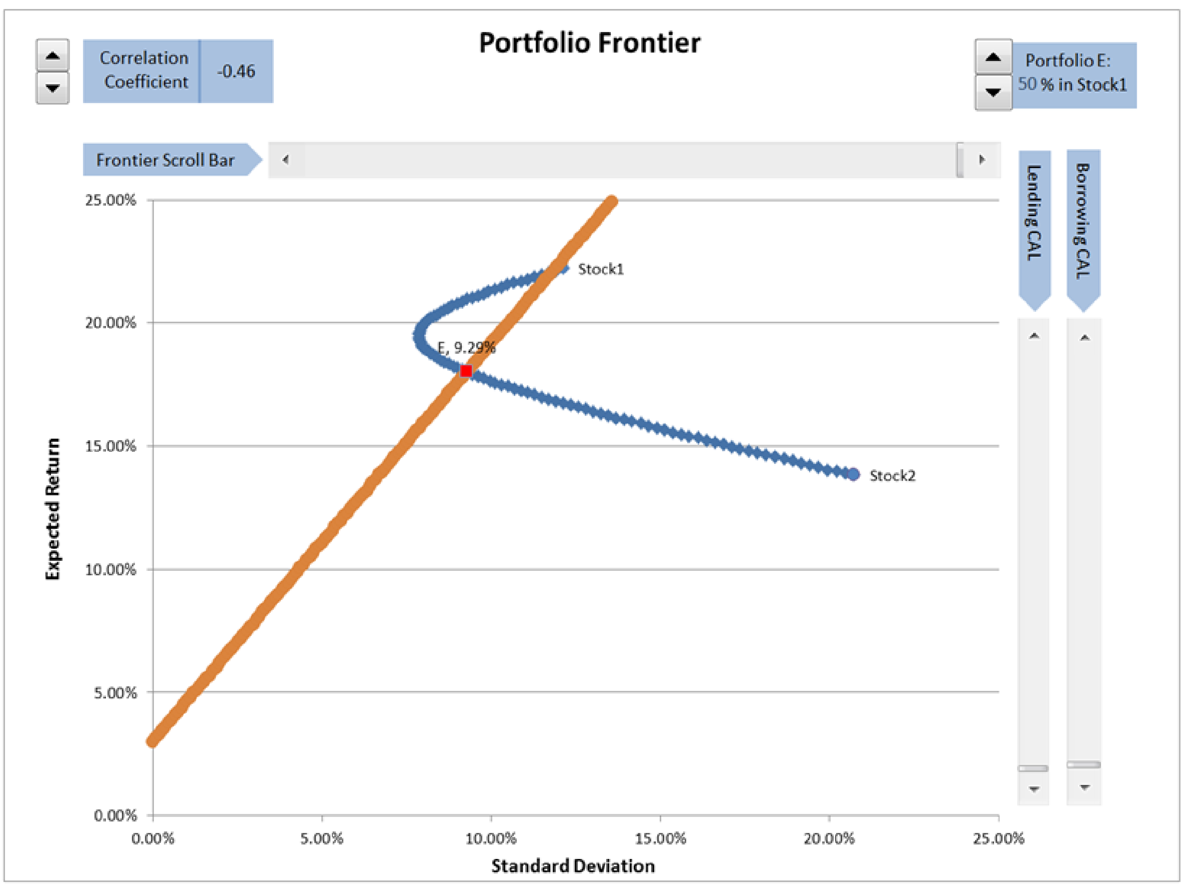
2. As with the Frontier Scroll Bar, the values to be entered include the minimum value, maximum value, incremental change and cell link. Input "0" as the minimum value (the lowest weight that can be placed into risky portfolio E) and "100" as the maximum value (the highest weight that can be placed into risky portfolio E). Input "1" as the incremental change--the amount by which the scroll bar value will change when the arrows on either end are pressed. You may wish to use a larger increment although an incremental value of "1" allows us to fine tune the tangency point weighting scheme of the Lending CAL and the efficient portion of frontier when lecturing on this issue.

3. Link the scroll bar value to N7 on the Data worksheet by inputting "Data!$N$7" in the Cell Link section.

4. Select "OK," deselect the scroll bar then scroll the Lending CAL Scroll Bar to test it. As the Lending CAL Scroll Bar is incremented to a higher value, the counter in Data!N7 changes and the values in H7:I107 are changed from #N/A to the SD and ER of the combination of investing in the riskless asset and risky portfolio E.

5. Add an arrow with the text "Lending CAL" to enhance the graph as shown in Figure 4 by choosing the Insert ribbon, then "Shapes" and the "Pentagon" from the "Block Arrows" section. Once you place the pentagon on the chart, select it and rotate it 90 degrees to the right by using the mouse to "grab" the green dot that appears at the top of the pentagon after you select it. Input the appropriate text (right click the pentagon and choose "Edit Text" to add text).

Figure 4. Risky Portfolio Frontier and Capital Allocation Line.



As of yet, the data points in H7:I107 have not been added to the graph. Before we add the data points to the graph, we have chosen to extend the CAL to incorporate riskless borrowing as well as lending. For this spreadsheet we assume a maximum investment in the risky portfolio E of 150% of one's wealth. Thus, we will add another 50 rows to column E on the Data worksheet, extending the weight from 100% to 150%.

1. Select E106:E107 and use the fill handle to drag the formula to E108:E157 to create the weights of 101% to 150%.

2. Highlight F107:G107 and use the fill handle to drag the formula to F108:G157. These cells extend the Lending CAL to the Borrowing CAL.

3. Highlight H107:I107 and use the fill handle to drag the formulas to H108:I108. H108:I108 represent the beginning of the range used to graph the Borrowing CAL. However, we prefer to use a separate scroll bar to graph the borrowing portion of the CAL. This scroll bar will need its own counter cell as N7 is used by the Lending CAL Scroll Bar. Thus, rather than referring to N7 in H108:I108, we need to refer to a new counter cell. We'll use O7 as the counter cell for the Borrowing CAL Scroll bar.

4. Edit the formulas in H108 and I108, changing the "$N$7" to "$O$7" in both cells.

5. Highlight H108:I108 and use the fill handle to drag the formulas to H109:I157.

To add the Borrowing CAL Scroll Bar to the graph and attach the value of the scroll bar to the counter cell O7:

1. From the Frontier worksheet select the Developer ribbon and "Insert" a "Scroll Bar (Form Control)." Use the mouse to size and position the scroll bar to the right of the Lending CAL Scroll Bar as shown in Figure 4.

2. Right click the scroll bar and select "Format Control."

3. Enter a minimum value of "101" (the minimum weight in the risky portfolio E if one is to be a borrower), a maximum value of "150" (the maximum weight we have chosen for a person borrowing to invest in risky portfolio E), an incremental change of "1" and set the cell link to "Data!O7." 4. Select OK to exit the "Format Control" dialog box.

5. Add an arrow with the text "Borrowing CAL" to enhance the graph as shown in Figure 4 by selecting the Chart Area. From the Insert ribbon, choose "Shapes" and "Pentagon" from the "Block Arrows" section and input the appropriate text (right click the pentagon and choose "Edit Text" to add text). Rotate the pentagon 90 degrees to the right as described earlier.

It is now time to add the data points for the Lending CAL and Borrowing CAL to the graph. Although two separate scroll bars control the lending and borrowing weights of the riskless asset, only one new series of data points is needed on the graph.

1. From the Frontier worksheet, right click in the Chart Area and choose "Select Data." Choose "Add" and input "Data!H7:H157" for the Series X values and "Data!I7:I157" for the Series Y values. Leave the "Series Name" blank.

2. Select "OK" and "OK" to close the dialog boxes. Test the Lending CAL Scroll Bar and the Borrowing CAL Scroll Bar to ensure that the links are functioning properly. Note that it looks awkward having the Borrowing CAL Scroll Bar at any value other than the minimum value if the Lending CAL Scroll Bar is not at its maximum value.

In classroom presentation, we explore different weights for the riskless asset and portfolio E to show different points on the CAL. The instructor can then ask students some leading questions, such as:

- Is there a better point (line) than the 50/50 weighting of STOCK1 and STOCK2?
- If so, how do you get there? [Specifically, should the weight of STOCK1 change (increase or decrease)?]
- Will investing 100% in the highest-return stock accomplish the best point?

By changing the weight of STOCK1 in P4 we can incorporate the students' responses. (NOTE: We perform this portion of the demonstration with all the scroll bars at their maximum values.) However, as P4 is on a separate worksheet from the chart, adding a spinner to the chart to control the weight of STOCK1 in Data!P4 will enable us to easily adjust the value of P4.

Spinner Control for Weight in STOCK1 of Risky Portfolio E

Just as we added a spinner to control the value of the correlation coefficient, adding a spinner to control the risky portfolio E enhances the visual presentation. We'll also add a textbox showing the weight in STOCK1 on the graph.

1. In the Frontier worksheet, choose Insert from the Developer ribbon and select "Spin Button" (Form Control). With the mouse place and size the spin button in the upper right of the chart. Right click the spinner and select "Format Control." The minimum and maximum values need to represent the possible range for the weight in STOCK1, which can range from 0% to 100%. Set the minimum value to "0," the maximum value to "100," the incremental change value to "1," and the cell link to "Data!P5."

2. The spinner places a value ranging from 0 to 100 in Data!P5. However, the weight should be a decimal amount rather than an integer. To convert the spinner range to a decimal, we can divide the spinner value by 100 (giving us a minimum value of 0 and a maximum value of 1). We need to replace the original STOCK1 weight in P4 with the converted value generated by the spinner. To do so, on the Data worksheet enter the following equation in P4:
=P5/100

Set all the scroll bars to their maximum position and note how the slope of the CAL changes as you adjust the spinner on the graph. Next, we'll add a text box to display the current weight of STOCK1 in risky portfolio E.

3. On the Frontier worksheet, select "Text Box" from the Insert ribbon. Using the mouse, draw a text box next to the spinner. Place your cursor in the formula bar (the area to the right of the fx) and input "=Data!P5." The current value for the spinner should appear in the text box. Add another

text box with the text "Portfolio E: % in STOCK1" as shown in Figure 4.

By interacting with the students as described above, we can lead students to the conclusion that the tangency point on the efficient portion of the frontier will produce an optimal CAL and that there is some optimal weighting of STOCK1 and STOCK2 that will achieve that point. While it is possible to determine the tangency point in Excel via the Solver function and VBA code, we generally have the students determine the tangency point by "eyeballing" it.

SUMMARY

The spreadsheet and interactive graph we present here strike a nice compromise between lengthy numerical calculations and superficial presentation. They allow the instructor to show how the frontier is generated and the effect of changing weights and/or the effect of different correlations. The demonstration also introduces the use of a riskless asset combined with borrowing and lending as a precursor to the idea of the Capital Market Line and capital market theory. We have found that students can grasp these concepts quickly and respond to questions easily when the interactive version is presented to them.

There are some obvious extensions that can enhance this spreadsheet:

1. Perform an interactive web query that will look up the prices for two stocks from some convenient website such as finance.yahoo.com and use that price data to calculate historical returns for estimating the inputs. (See Carter, et al, [2002] for one way to accomplish this).
2. Extend the spreadsheet to more than two stocks.

ENDNOTES

1 The paper assumes the reader has a basic understanding of Excel, including the difference between absolute and relative references/notation, and basic graphing using XY Scatter Plots.

2 The Capital Allocation Line connects any point on the frontier with the riskless asset. If the frontier were generated from all risky assets, the line extending from the riskless asset to the tangency point on the efficient portion of the frontier would be the well-known Capital Market Line.

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Using Excel Spreadsheeets to Teach the Time Value of Money

Richard Followill and Brett Olsen

Possibly the most important precursor of academic success for students of finance is an understanding of the mathematics of interest and the time-value of money. Although a considerable portion of their first finance course is usually devoted to this topic, students often proceed to their advanced courses with inadequate knowledge of this vital subject. In this article, we present and make available Excel spreadsheet templates that provide students with a nearly unlimited collection of problems and solutions to help them efficiently perfect their time-value skills.

INTRODUCTION

In an earlier issue of this journal (Followill, 2010) we described our efforts to instill in our Finance and Real Estate majors, through a series of extracurricular seminars, the time-value of money problem-solving skills they will assuredly need in their future careers. This effort has been quite successful, resulting in enhanced learning in our upper-level finance courses, making them far more enjoyable for our students and their professors alike.

In order to entice our students to voluntarily engage in a series of late afternoon seminars that promises to be hard work for zero academic credit, we decided to provide all materials at no cost, and to recognize our majors who passed a difficult exam with only one or no errors. But we knew it was even more important to present to our students a learning experience that *they* found to be valuable. We realized that if the seminars and provided materials were not designed and delivered in an efficient and effective manner, word of mouth among our majors would surely end our project at its very beginnings. Fortunately, our students delivered a positive verdict, and the Mathematics of Interest Seminars have been held each semester at the University of Northern Iowa for the past seven years.

Our methods, as we recount them in the 2010 article, rely on an equations-oriented, rule-based approach that requires our majors to understand the underlying mathematics of time-value. We guide our students through the development of the time-value formulas, solve numerous problems during the lectures, and provide written versions of unsolved and solved problems. We also rely heavily on the use of Excel templates that generate a nearly unlimited set of problems and solutions in the different areas of time-value.

The purpose of this article is to present these time-value templates and, more importantly, make them available for others to use via the *Journal of Informational Techniques in Finance* website, http://www.jfcr.org/jitfvols.html.

THE TEMPLATES

We divide the Excel time-value templates into four categories:

1) The present and future value of a lump sum amount,
2) Annuities and perpetuities,
3) Interest rate manipulation, and
4) Complex problem solving.

In each of the categories we provide an Excel template that presents students with a nearly infinite collection of problems to solve. Hints, formulas, cash flow diagrams, time-value rules, and solution procedures are also provided within the templates. Students are encouraged to drill themselves and check their solutions and thought processes against the solutions provided for each problem.

The present and future value of a lump sum amount; introducing the "building-block" equation

We refer to Equation 1 as the *building-block equation* because everything in the mathematics of time-value derives from it: the equations for annuities and perpetuities, and the procedure for manipulating interest rates.

$$FV = PV(1+i)^n \tag{1}$$

where FV signifies future value, *PV* is present value, *i* is a simple interest rate across a period defined by *n*, and *n* represents the number of periods. The template generates questions that require students to solve *PV, FV, i,* or *n*. For some problems, *i* is an annual rate compounded annually, but for other problems, *i* will be an annual rate compounded other than annually. Figures 1 and 2 present the template problems and solutions for solving for *PV* and *FV*.

Figure 1. Sheet 1 of Template 1; *PV* and *FV* Problems

The Time Value of Money Templates ***Template 1: PV and FV of a Lump Sum***

Rule 1: The interest rate, *i*, and the period defined by *n* must correspond.

Find the Present Value of a Lump Sum

PROBLEM: What is the value today of $8,142.00 to be received at the end of 16 years if the annual interest rate is 8.10% compounded weekly?

HINT: Use the BUILDING BLOCK EQUATION $PV = \frac{FV}{(1+i)^n}$

Find the Future Value of a Lump Sum

PROBLEM: If you deposit $4,809.00 in an account today that pays 4.80% compounded quarterly, how much will you have in 10 years?

HINT: Use the BUILDING BLOCK EQUATION $FV = PV(1+i)^n$

The solutions are provided in the 'PV-FV Solutions' tab.

For new problems, press 'F9'.

Figure 2. Sheet 2 of Template 1; *PV* and *FV* Solutions

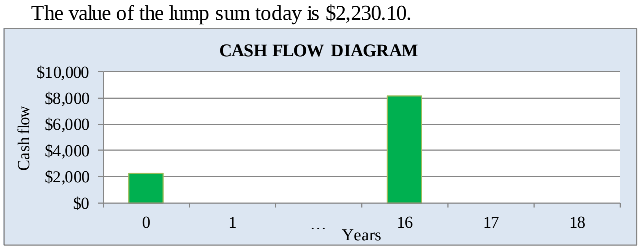
The Time Value of Money Templates

Find the Present Value of a Lump Sum

PROBLEM: What is the value today of $8,142.00 to be received at the end of 16 years if the annual interest rate is 8.10% compounded weekly?

Solution:

The value of the lump sum today is $2,230.10.



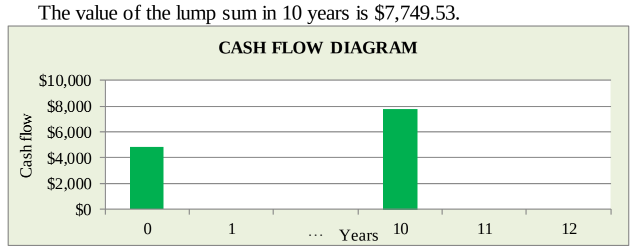
Since the annual interest rate is 8.10% compounded weekly, be sure to express i in weekly periods.

Find the Future Value of a Lump Sum

PROBLEM: If you deposit $4,809.00 in an account today that pays 4.80% compounded quarterly, how much will you have in 10 years?

Solution:

The value of the lump sum in 10 years is $7,749.53.



The same principle applies to this problem as well. REMEMBER, i and n MUST correspond.

When we teach the material presented in these templates we stress the first of our time-value rules:

Rule 1: The interest rate, *i*, and the period defined by *n* must correspond.

If, for example, *i* is a nominal (or stated) annual rate compounded monthly, the number of periods, *n*, must be expressed in months. Understanding this rule is vital to correctly solving for *i* and *n* as shown in figures 3 and 4.

Each template is designed so that a new set of questions is presented each time the student presses F9. The number of individual questions that can be generated is not infinite, but the probability that a student will see the same question twice is quite small.

Once students have attained a complete understanding of the building-block equation, they are ready to tackle annuities and perpetuities.

Figure 3. Sheet 3 of Template 1; Solving for *i* and *n*.

The Time Value of Money Templates ***Template 1: Solving for i and n***

Find the Interest Rate

PROBLEM: Suppose you want to have $5,610.00 in 12 years. You currently have $2,805.00 to invest. What annual interest rate must you earn on your investment to reach your goal? Assume quarterly compounding.

Find the Number of Periods

PROBLEM: You have set a goal to have $3,426.00. You currently have $1,645.00 to invest. An account promises 3.00% interest, compounded semi-annually. How long, in years, will it take for you to reach your goal?

HINT: Use the BUILDING BLOCK EQUATION ==> $FV = PV(1+i)^n$

Remember, i and n must correspond; if i is compounded monthly, n must be expressed in months, etc. Find the rate or number of periods using this rule, then convert to an annual rate, or annual periods.

The solutions are provided in the "i and n solutions" tab.

For new problems, press 'F9'.

Figure 4. Sheet 4 of Template 1; *i* and *n* Solutions.

The Time Value of Money Templates

Find the Interest Rate

PROBLEM: Suppose you want to have $5,610.00 in 12 years. You currently have $2,805.00 to invest. What annual interest rate must you earn on your investment to reach your goal? Assume quarterly compounding.

Solution:

The annual interest rate is 5.82 % compounded quarterly.

Since interest is compounded quarterly, then n must be expressed in quarterly periods. Now, convert to a stated annual rate compounded quarterly.

Find the Number of Periods

PROBLEM: You have set a goal to have $3,426.00. You currently have $1,645.00 to invest. An account promises 3.00% interest, compounded semi-annually. How long, in years, will it take for you to reach your goal?

Solution:

The number of periods is 24.64 years.

Since interest is compounded semi-annually, your solution for n will be in semi-annual periods. Now convert to years.

Annuities and perpetuities

Our second template introduces our students to annuities and perpetuities. We present only the equations for ordinary annuities at this point.
The equation for the present value of an annuity is:

$$PVA = pmt\left(\frac{1-\frac{1}{(1+i)^n}}{i}\right) \tag{2}$$

where *pmt* is the annuity payment. While *n* is still the number of periods for an ordinary annuity, we stress that *n* is *always* the number of annuity payments.

The equation for the future value of an annuity is

$$FVA = pmt\left(\frac{(1+i)^n-1}{i}\right) \tag{3}$$

The equation for the present value of a perpetuity is:

$$PV_\infty = \frac{pmt}{i} \tag{4}$$

The rules we stress for ordinary annuities represented by the above equations are:

Rule 2: Pay attention to the time-line. The *PVA* of an annuity equation (or perpetuity equation) consolidates the stream of equal payments into a lump sum amount *one period* before the first payment is made. The *FVA* of an annuity equation consolidates all payments at *the point in time the last payment is made.*

Rule 3: For annuities, *n* is the number of payments—***ALWAYS***.

Figure 5. The Interest Rate Manipulation On/Off Switch.

Figure 6. Sheet 1 of Template 2; *PVA* and *FVA* Problems.

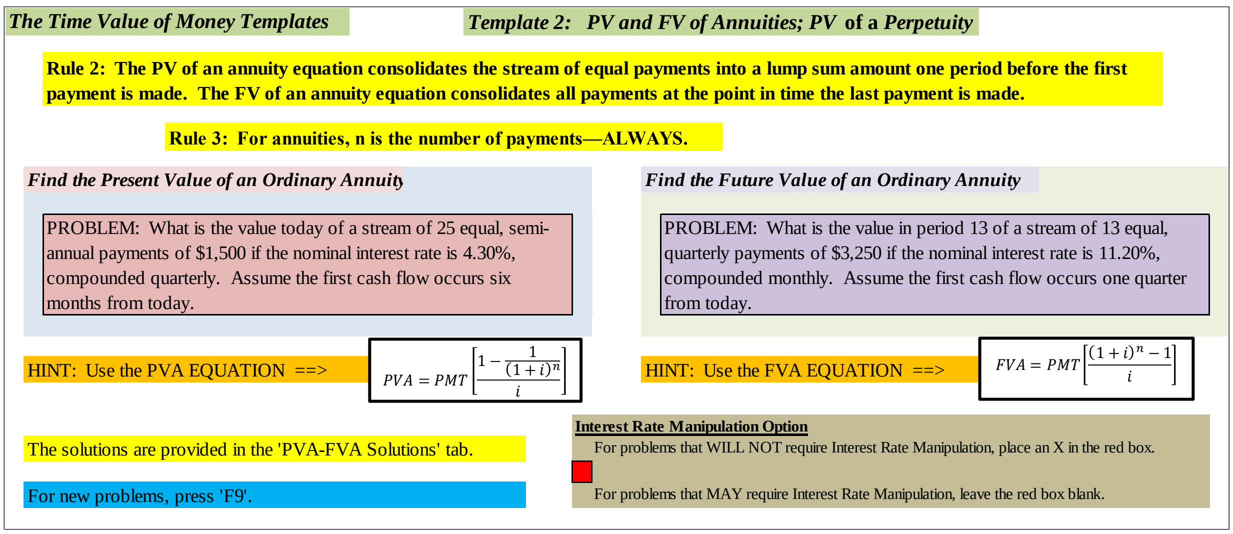
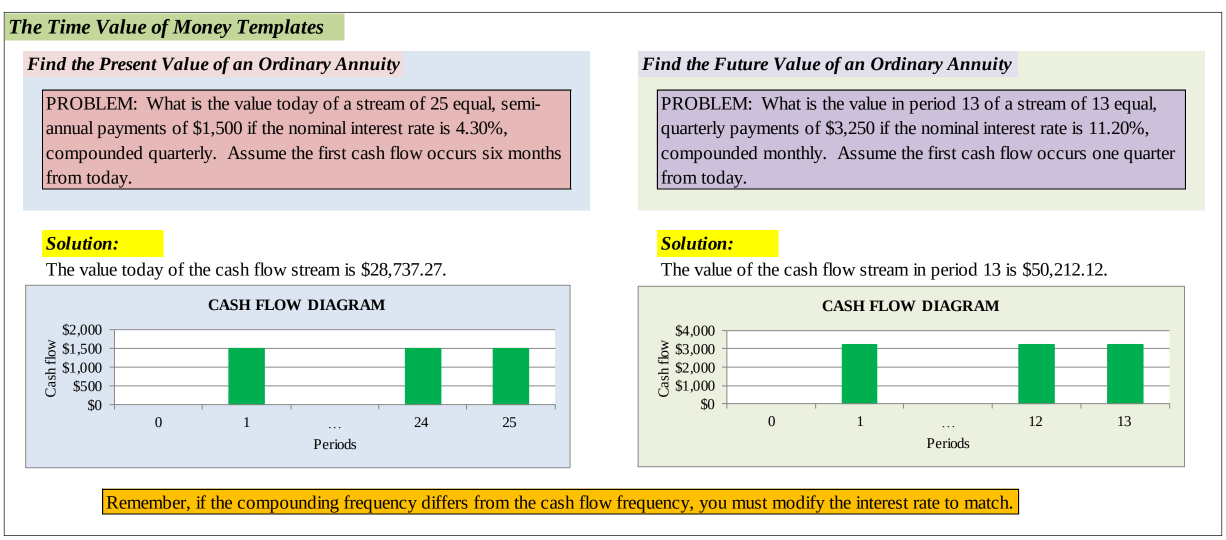


Figure 7. Sheet 2 of Template 2; *PVA* and *FVA* Solutions



When we first introduce annuities to our students, we use the version of our second template that is designed to produce problems where the given interest rate fits exactly the time between payments. After we introduce interest rate manipulation in our third template we encourage students to return to the annuities template and switch to the version that requires interest rate manipulation. Figure 5 shows the switch that appears in Template 2 and in Template 4, the complex problems template.

Figures 6 and 7 show the problem and solution sheets for the *PVA* and *FVA* of ordinary annuities. Notice that in figure 6, the interest rate manipulation option is in force and the given interest rate does not fit the time span between payments. Students will have to manipulate the given interest rate to find an effective rate consistent with the time between payments.

Sheets 3 and 4 of Template 2 present problems and solutions for the present value of a perpetuity. Since these pages are similar to those shown in figures 6 and 7, we do not present them here.

Interest rate manipulation

Interest rate manipulation may be necessary in order to compare two rates with different compounding periods, or to calculate an effective interest rate for use in the annuity and perpetuity equations. The equation for calculating an effective interest rate across any time span is:

$$EffectiveRate = \left(1 + \frac{Stated.Annual.Rate}{Compounding.Periods.Per.Year}\right)^{\substack{Compounding.Periods \\ Across.the.Time.Span}} - 1 \qquad (5)$$

Figure 8. Sheet 1 of Template 3; Interest Rate Manipulation Problems

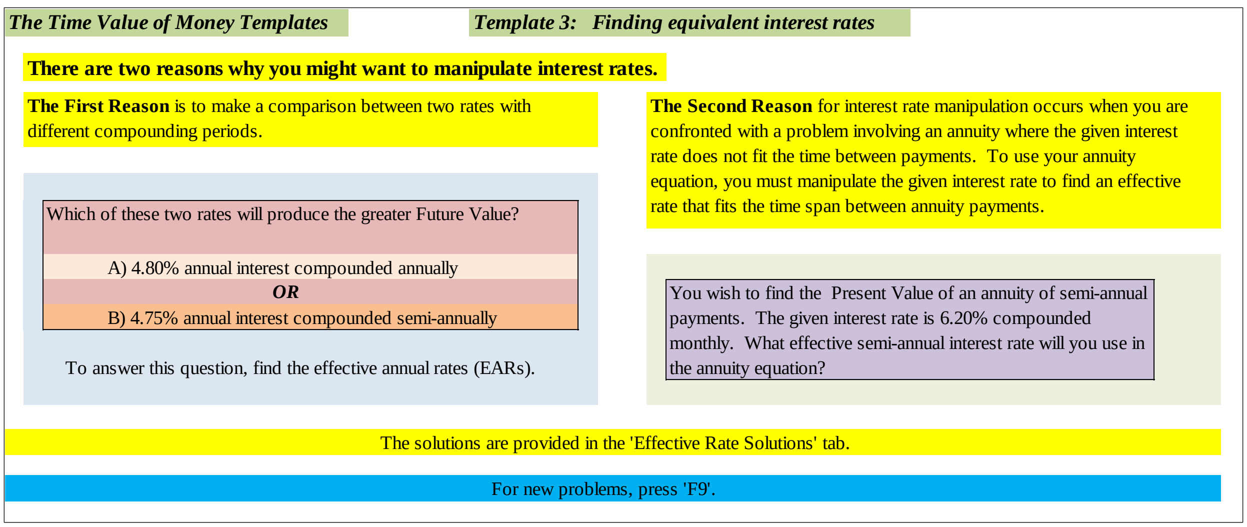
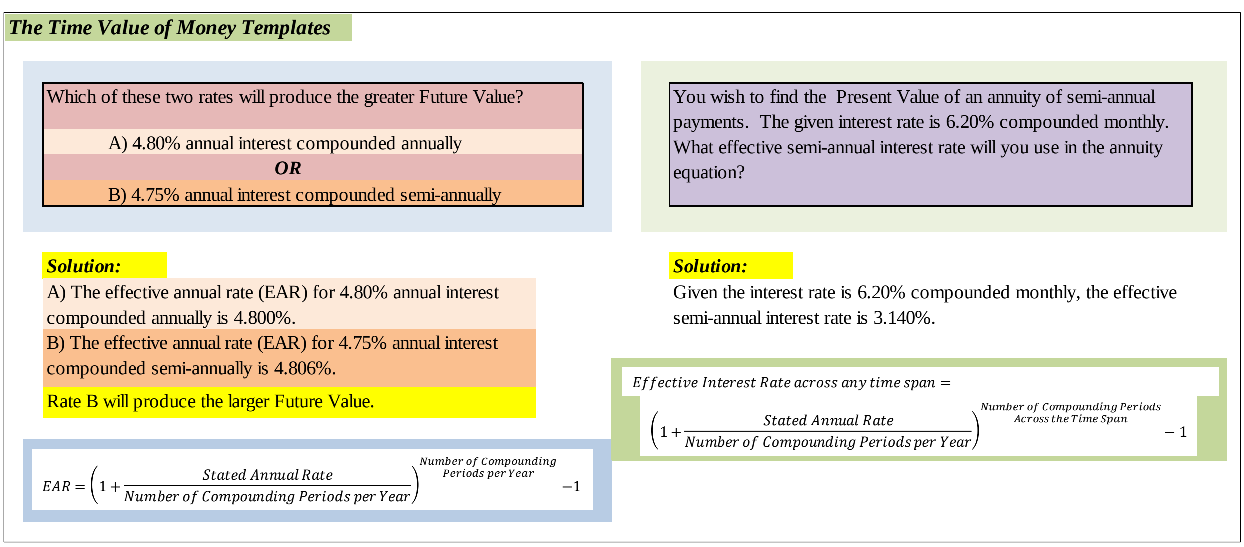


Figure 9. Sheet 2 of Template 3; Interest Rate Manipulation Solutions



Manipulating interest rates to fit a desired time period often proves to be a difficult concept for students to master. A key to understanding interest rate manipulation is to note the similarity between Equation 5 and the building block equation, $FV = PV(1+i)^n$, and the importance of Rule 1. In both Equation 1 and Equation 5, the interest rate and the period defined by n must correspond. Since students have now been introduced to annuities, we expand Rule 1 as follows:

Rule 1 (addendum): *i* and *n* must correspond. For the building block equation, the period for i is given, and n defines the number of periods between PV and FV. For annuities, the appropriate periodic rate should be used. Since the time between payments is given, i may have to be adjusted to fit the time period.

After some practice using the interest manipulation template, students are directed to return to the annuities and perpetuities template and engage the switch that requires interest rate manipulation in order to use the annuity equations.

Figures 8 and 9 present the problem and solution pages for interest rate manipulation.

Complex problem solving

Once students have mastered the basics of time-value, it is time to apply them to solving the complex problems presented in our final template. We follow the conventions of our previous templates, giving the student the option, when possible, of selecting problems that do or do not require interest rate manipulation. As with all of the templates, pressing F9 reconfigures each problem type with respect to the interest rate, the number and size of annuity payments, and timing of the cash flows.

A solution to each problem is presented on the adjacent sheet, along with a cash flow diagram, and a possible solution procedure. Problem-solving hints and the inviolate rules of time value that apply to each problem are also given as well.

We stress to students that there are usually several straightforward methods to arrive at the correct solution to any time-value problem, and that *the key to solving complex time-value problems* is to move all the money to a singular point in time. Once that is accomplished, the solution for the unknown, whether it is a lump sum amount, the payment size, or the interest rate, should be readily apparent.

For success in solving complex time-value problems, we stress our final time-value rule.

Rule 4: NEVER compare amounts of money unless they are at the same point in time.

The collection of complex problems is extensive and students who can work all of them should have little difficulty solving any time-value problem they may later encounter.

Figures 10 and 11 present just the first of the twenty complex problems found in our fourth template. Each problem is first presented on a question sheet. We encourage students to try to work the problem presented to them before moving to the next sheet which presents the solution.

Each problem is followed by a solution sheet that presents one possible solution procedure along with hints and problem solving suggestions. Each of the twenty problem formats has numerous permutations that may be accessed by pressing the F9 recalculation key.

Figure 10. Sheet 1 of Template 4; The First of Twenty Complex Time-Value Problems.

The Time Value of Money Templates ***Template 4: Complex problems***

Q1

PROBLEM: You have decided to make equal, annual deposits of $602.00 to an account that pays 9.00% compounded monthly. You plan to make a total of 12 deposits. The first deposit will be made today and the final deposit will be made at the end of year 11.

At the end of year 11, you will make the first of 10 annual withdrawals of $662.00. How much money will remain in the account at the end of year 20 when the final withdrawal is made?

Interest Rate Manipulation Option

For problems that WILL NOT require Interest Rate Manipulation, place an X in the red box.

For problems that MAY require Interest Rate Manipulation, leave the red box blank.

The solutions are provided in the 'A1' tab.

For new problems, press 'F9'.

Figure 11. Sheet 2 of Template 4; A Solution to the First Complex Time-Value Problem

The Time Value of Money Templates

A1

PROBLEM: You have decided to make equal, annual deposits of $602.00 to an account that pays 9.00% compounded monthly. You plan to make a total of 12 deposits. The first deposit will be made today and the final deposit will be made at the end of year 11.

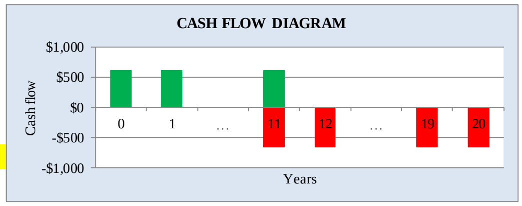
At the end of year 11, you will make the first of 10 annual withdrawals of $662.00. How much money will remain in the account at the end of year 20 when the final withdrawal is made?

HINT: You need to determine a value in the FUTURE. Thus, transform all cash flows into Future Values at the end of year 20.

$$FVA = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

Solution:

The effective annual interest rate is 9.381%.
The value of the deposits in year 11 is $12,403.86.
The value of the deposits in year 20 is $27,798.59.
The value of the withdrawals in year 20 is $10,242.30.
The amount of money remaining in year 20 is $17,556.29.



Note to Students: The computer does not round in its internal calculations. The rate of 9.38% shown at left is actually 9.38069%. If you use 9.38%, your answer will be slightly lower than the computers answer even though your procedure is correct.

CONCLUSION

As the reader has already ascertained, we take a minimalist approach to teaching time-value to our students. We do not introduce some fairly basic topics such as annuities due, or conventions such as sinking fund loans, and we eschew more advanced topics such as growing annuities and perpetuities, gradient or step function annuities, or continuous compounding. Our entire motivation is to provide students with a basic understanding of time-value so that when they encounter topics such as the dividend growth model, duration, and the option pricing models in their higher level finance classes, they are quickly able to understand their presentation and grasp their meaning. When students have achieved this level of competence, teaching becomes much easier and vastly more rewarding.

We strongly believe in taking an equations-oriented, rule-based approach to teaching the mathematics of interest and the time-value of money, and we look upon the financial calculator-based instruction we see in many textbooks with some dismay. Financial calculators are useful, time-saving devices, and nearly all of our students who become adept at time-value learn to use the TVM buttons on their own. Almost invariably, however, when we put a time-value problem on an exam along with the usual admonishment, "show your work," and receive drawn pictures of the buttons and entries used to solve the problem, the answer is wildly incorrect.

At the University of Northern Iowa we have had measurable success instilling the basics of the time-value of money in our students. We gladly make these Excel templates available for other finance professors to use and improve upon. The Excel spreadsheets are available at www.jfcr.org. Comments regarding the templates are appreciated and may be directed to richard.followill@uni.edu and brett.olsen@uni.edu.

REFERENCE

Followill, R. (2010) *Time Value of Money Seminars: Immersion Therapy for Finance Majors*, Journal of Instructional Techniques in Finance, 2:1 9:12.

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